MATH 165
FALL 2012
EXAM 3

1. (36 pt) Evaluate the following limits:

a) $\lim_{x\to 0}(1+ax)^{\frac{b}{x}}$ b) $\lim_{x\to -\infty}(x-(x^3+1)^{\frac{1}{3}})$

c) $\lim_{x\to 1}\sqrt[3]{\dfrac{x^7+x^6+2x-4}{3x^3+x^2-4}}$

2. (15 pt) Sketch the graph of $f(x)=x-(x^3+1)^{\frac{1}{3}}$. The first two derivatives are given by:

$$f'(x)=\frac{(x^3+1)^{\frac{2}{3}}-x^2}{(x^3+1)^{\frac{2}{3}}}$$

and

$$f''(x)=\frac{-2x}{(x^3+1)^{\frac{5}{3}}}.$$

3. (15 pt) Sketch the graph of $f(x)=\frac{x^{\frac{1}{3}}}{\ln(x^2)}$

$$f'(x)=\frac{x^{\frac{1}{3}}(\ln(x^2)-6)}{3(\ln(x^2))^2x}$$

and

$$f''(x)=\frac{-2x^{\frac{1}{3}}((\ln(x^2))^2-3\ln(x^2)-36)}{9(\ln(x^2))^3x^2}.$$

(the roots of $(\ln(x^2))^2-3\ln(x^2)-36$ are approximately $\pm.1$ and ± 46.1.)

4. (15 pt) A billboard is b feet wide and is perpendicular to a long straight highway and is a feet from the road. How far from the point closest to the billboard on the highway are you when you get the best look at the billboard?

5. (15 pt) The surface area of a cone (not including the base) is given by $S=\pi r\sqrt{r^2+h^2}$, where r is the radius of the cone and h is the height. Find the cone of maximal surface area (again, not including the base) that can be inscribed in a sphere of radius R.

6. (6 pt) Consider the function $f(x)=x^3-3x$. Find the positive real number, z, such that the next Newton's method approximation is $-z$. What happens if you let z be your first approximation?

7. (8 pt) Find a formula for the n^{th} derivative of $f(x)=xe^{ax}$, $a\neq 0$ and prove that the formula works by induction (take a few derivatives to establish the pattern).