MATH 165
EXTRA CREDIT PROBLEMS

1. In class we noticed that if $f(x)$ is either (always) increasing or (always) decreasing, then it is one to one. Is the converse true? That is, if $f(x)$ is 1-1 then is it either always increasing or always decreasing (prove or give a counterexample)? What if $f(x)$ is **continuous** and 1-1; is it always increasing or always decreasing?

2. In class we pointed out that the notation $f^{-1}(x)$ **does not mean** $\frac{1}{f(x)}$ (please remember this!). Can you give an example of a function $f(x)$ such that

$$f^{-1}(x) = \frac{1}{f(x)}?$$

The more examples that you can give the better. (Can you list all functions with this property? Can you give more than 2 examples of functions with this property?)

3. Give an example of a function that is differentiable on all of $\mathbb{R}$, but its derivative is not continuous.

4. One of the related rates problems in the homework (or from class) describes a 13 foot ladder that is leaning against a wall. The ladder begins to slide so that the bottom of the ladder moves away from the wall at a constant speed of 2 feet per second and asks how quickly the top is sliding down the wall when it is 5 feet from the floor.

a) Work this problem out and show/explain why that although it is the case that this model (probably) works well for the problem at hand (that is, when the top of the ladder is 5 feet above the ground), it cannot possibly work throughout the ladder's entire trip to the floor.
b) In the real world, the ladder will not stay in contact with the wall and will eventually come crashing down. Find out when the model "falls apart" (that is, predict when the ladder must leave the wall assuming that its bottom moves at a constant 2 feet per second).

5. You are most apt to spill your can of Coke when it is full or when it is empty (in both of these cases the center of mass of the can of Coke is the "dead center" of the can). When is the can the most stable (that is, describe the situation in which the center of mass of the can of Coke is at its lowest).

6. Let $f(x)$ be a function.

a) Show that the graph of $f(x)$ has the line $y = mx + b$ as a slant asymptote to the right if and only if $\lim_{x\to\infty} \frac{f(x)}{x} = m$ and $\lim_{x\to\infty}(f(x) - mx) = b$.
b) Find and verify similar conditions for the graph of $f(x)$ to have a slant asymptote to the left.
c) Give an example of a continuous function (that is not piecewise linear) that has two different slant asymptotes to the left and to the right.

7. Use some geometry and what you have learned about definite and indefinite integrals to determine the antiderivative of the function $f(x) = \sqrt{R^2 - x^2}$. In particular, find

$$\int_0^x \sqrt{R^2 - t^2}\; dt.$$

Note: This particular problem has a shelf life and trigonometric substitutions are not allowed.