MATH 165
FALL 2003
FINAL EXAM

1. (20 pt) Evaluate the following limits if they exist.

a) $\lim_{x\to 2} \frac{x^5-32}{x+2}$ b) $\lim_{x\to 0} \frac{\sin(bx^2)}{e^{ax^2}-1}$ where $a \neq 0$ c) $\lim_{x\to 0}(ax+1)^{\frac{b}{x}}$

d) $\lim_{x\to\infty} \frac{x^2\cos(x)}{x^4+1}$ e) $\lim_{x\to -\infty} \frac{\sqrt[3]{x^3+1}}{\sqrt{4x^2+1}}$

2. (24 pt) Find the derivatives of the following functions.

a) $f(x) = (\sin(x))^{\cos(3x)}$ b) $g(x) = \frac{x\tan(2x)}{\sqrt[3]{x+1}\sqrt{2x-1}}$ c) $h(x) = \tan^{-1}(\sin(x)\cos(x)\ln(x))$

d) $k(x) = \frac{\sin(e^{x^2})}{2x^2+1}$

3. (18 pt) Evaluate the following integrals.

a) $\int_2^5 \frac{x^2+3x+4}{x+1}dx$ b) $\int_0^{\ln(2)} \frac{e^x}{e^{2x}+1}dx$ c) $\int \frac{(\ln(x))^n}{x}dx$, where $n \neq -1$.

4. (6 pt) Use the definition of the derivative to find the derivative of $f(x) = \sqrt{ax+b}$ with $a \neq 0$.

5. (6 pt) Let $g(x)$ be the inverse function of $f(x) = x^5 + x + 3$. Find the derivative of the function

$$F(x) = \int_0^{f(x)} g(t)dt.$$

6. (6 pt) Suppose that a spherical comet begins to evaporate as it approaches the sun. Suppose that it evaporates in such a way that its surface area decreases at the constant rate of 10 square feet per minute. Find the rate at which its volume is decreasing when the radius of the comet is 50 feet.

7. (8 pt) Find the largest surface area of a cylinder with no top or bottom that can be inscribed in a sphere of radius R.

8. (8 pt) Sketch the graph of the curve $f(x) = \sqrt[3]{\frac{x+1}{x+2}}$. The first two derivatives of this function are $f'(x) = \frac{1}{3(x+1)^{\frac{2}{3}}(x+2)^{\frac{4}{3}}}$ and $f''(x) = \frac{-(6x+8)}{9(x+1)^{\frac{5}{3}}(x+2)^{\frac{7}{3}}}$.

9. (3 pt) Let $f(x)$ be a function that is continous on $[a, b]$. State the definition of the definite integral of $f(x)$ from a to b.

10. (5 pt) You are computing the surface area of a circular lake and you find that the radius is 10 miles. Later a surveyor comes in and tells you that the original measurement of 10 miles could have an error of $\pm\frac{1}{10}$ of a mile. Use differentials to estimate the error in using the original figure of 10 miles to compute the area. Also use differentials to estimate the relative error.

11. (6 pt) Show that the function $f(x) = x^3 + x + \frac{1}{3}\cos(2x)$ has one and only one root.