MATH 165
FALL 2004
FINAL EXAM

1. (25 pt) Evaluate the following limits if they exist.

a) $\displaystyle\lim_{x\to 0}\frac{\sin(ax)}{\tan((b^2+1)x)}$ b) $\displaystyle\lim_{x\to 3}\ln\left(\frac{x^3-27}{x^2+2x-15}\right)$ c) $\displaystyle\lim_{x\to\infty}\frac{\sqrt[3]{2x^3+3x+4}}{1-2x}$

d) $\displaystyle\lim_{x\to\infty}\frac{f(x)}{x}$ where $mx+a\le f(x)\le mx+b,\ a<b$ e) $\displaystyle\lim_{x\to 0}(1+ax)^{\frac{b}{x}}$

2. (24 pt) Find the derivatives of the following functions.

a) $f(x)=\dfrac{k(x)m(x)}{g(x)}$ b) $g(x)=x^2\ln(|\sec(x^2+3x)|)$ c) $h(x)=(2x)^{e^x}$

d) $k(x)=\sin(\cos(\ln(e^{2x}+1)))$

3. (18 pt) Evaluate the following integrals.

a) $\displaystyle\int 2x^3\sqrt{x^2+1}\,dx$ b) $\displaystyle\int_0^{\frac{\pi}{2}}\frac{\cos(x)}{\sin^2(x)+1}dx$ c) $\displaystyle\int_0^{\ln(3)}\frac{e^x(e^{2x}+1)}{e^x+1}dx$

4. (5 pt) Use the definition of the derivative to find the derivative of $f(x)=\frac{1}{\sqrt{x}}$.

5. (5 pt) Use the definition of the definite integral to find $\int_0^2(3x^2+3)dx$.

6. (8 pt) A water tank is in the shape of an inverted cone of height 10 meters and radius of 4 meters. The tank has sprung a leak and you find that the depth of the water in the tank is decreasing at $\frac{1}{10}$ meters per hour when the depth of the water is 5 meters. How fast is the tank leaking?

7. (8 pt) Farmer Stu has P feet of fence. He wishes to make two adjacent rectangular pens (sharing a border) of equal area with this fence. Describe how to make these pens to maximize the total area.

8. (8 pt) Sketch the graph of the curve $f(x)=4x^{\frac{1}{3}}+x^{\frac{4}{3}}$. For your convenience, the first two derivatives of this function are $f'(x)=\frac{4}{3}x^{-\frac{2}{3}}(1+x)$ and $f''(x)=\frac{4}{9}x^{-\frac{5}{3}}(x-2)$.

9. (5 pt) Find the tangent line to the function given implicitly by the equation $xy=\cos(2x-y)$ at the point $(0,\frac{\pi}{2})$.

10. (4 pt) Use a linear approximation to estimate the value of $\sqrt[4]{15}$. Is your answer an overestimate or an underestimate (and explain how you know)?