MATH 165
FALL 2005
FINAL EXAM

1. (20 pt) Evaluate the following limits if they exist.

a) $\lim_{x\to 0}\sin(ax)\cot(bx), b\neq 0$ b) $\lim_{x\to 1}\frac{x^6-4x^3+3}{x^4-2x^3+1}$ c) $\lim_{x\to\infty}(1+\frac{b}{x})^{cx}$

d) $\lim_{x\to\infty}\frac{|x|\cos(x)}{x^2+1}$

2. (24 pt) Find the derivatives of the following functions.

a) $f(x)=\frac{F(G(x))K(x)}{H(x)}$ b) $g(x)=\sin(x+\ln(\frac{x}{x+1}))$ c) $h(x)=x^{e^x}$

d) $k(x)=\int_{\cos(2x)}^{\ln(x)}\tan^2(t)dt$

3. (18 pt) Evaluate the following integrals.

a) $\int\frac{e^{2ax}}{e^{ax}+1}\,dx, a\neq 0$ b) $\int_0^2\frac{2x}{x^4+1}dx$ c) $\int(\frac{x^2+1}{x-1})^2dx$

4. (4 pt) Use the definition of the derivative to find the derivative of $f(x)=\sqrt{2x}$.

5. (5 pt) Use the definition of the definite integral to find $\int_1^3(4x+5)dx$.

6. (10 pt) A spherical star is collapsing under its own gravity. During this collapse, its radius is decreasing at 1000 miles per second. Find out how fast the surface area AND the volume of the star are changing when the radius of the star is 10000 miles? (The volume and surface area of a sphere are given by $V=\frac{4}{3}\pi R^3$ and $S=4\pi R^2$.)

7. (10 pt) A cylindrical can is to be constructed to hold 600 cubic inches. Find the minimal cost for constructing such a cylindrical can if the cost of constructing the top and bottom is 4 cents per square inch and the cost of the cylindrical “side” is 2 cents per square inch.

8. (10 pt) Sketch the graph of the curve $f(x)=4(x^2-1)^{\frac{2}{5}}$. The first two derivatives are given by

$$f'(x)=\frac{16x}{5(x^2-1)^{\frac{3}{5}}}$$

and

$$f''(x)=\frac{-16(x^2+5)}{25(x^2-1)^{\frac{8}{5}}}.$$

9. (4 pt) A curve is defined implicitly by the equation $x^3+y^3=x+y^2+1$. Find a formula for $\frac{dy}{dx}$.

10. (5 pt) We know that the tangent of a 45 degree angle is 1. Use differentials (or any linear approximation) to estimate the tangent of a 42 degree angle. Is you estimate too large or too small?