MATH 165
FALL 2007
FINAL EXAM

1. (20 pt) Evaluate the following integrals.

$$\text{a) } \int_{-1}^{2} \frac{x^3}{\sqrt{x^2+1}}dx \qquad \text{b) } \int e^x \sin(e^x)dx \qquad \text{c) } \int_1^{64} \frac{dx}{\sqrt{x}+\sqrt[3]{x}} \qquad \text{d) } \int \frac{ax}{bx+c}dx$$

2. (20 pt) Evaluate the following limits.

$$\text{a) } \lim_{x\to 1} \frac{x^3-2x+1}{x^4-3x^2+2} \qquad \text{b) } \lim_{x\to\infty} ax\tan(\frac{b}{x}) \qquad \text{c) } \lim_{x\to\infty} \int_0^x \frac{2t}{t^4+1}dt$$

$$\text{d) } \lim_{x\to-\infty} (\sqrt{x^2+7x} - \sqrt{x^2-3x+1})$$

3. (24 pt) Find the derivative for each of the following functions.

$$\text{a) } f(x) = F(\tan(e^{x^2}))G(\sin(x^2)) \qquad \text{b) } g(x) = \sqrt{\frac{(x^2+1)\sin(2x)}{(x^2+3)^{\frac{1}{3}}(5x+2)^5\ln(x)}} \qquad \text{c) } h(x) = (x^2+x+6)^{x^3\sin(2x)}$$

4. (6 pt) Use the definition of the definite integral to evaluate $\int_0^2 (3x^2+5)dx$.

5. (8 pt) Use the definition of the derivative to find the derivative of the function $f(x) = \sqrt{ax+b}$.

6. (6 pt) Suppose that a spherical snowball is put in an oven and it melts. Suppose that the snowball maintains its spherical shape throughout the melting process and that the puddle of water from the melted snow increases at the constant rate of 1 cubic inch per second. How fast is the surface area of the snowball decreasing when the snowball is a 1 foot in diameter?

7. (8 pt) Graph the function $f(x) = \tan^{-1}(\frac{1}{x^2-1})$. The first two derivatives of this function are $f'(x) = \frac{-2x}{x^4-2x^2+2}$ and $f''(x) = \frac{2(3x^4-2x^2-2)}{(x^4-4x^2+2)^2}$ (for your convenience the real roots of $3x^4-2x^2-2$ are approximately -1.82 and 1.82).

8. (8 pt) Find the maximum and minimum values of the function $f(x) = 2\sin(x) + \cos(2x)$ on the interval $[0, \frac{\pi}{2}]$.

9. (5 pt) Consider the function $f(x) = \int_1^{x^2} \frac{1}{t}dt$. If we denote y as the inverse function of $f(x)$, show that $y' = \frac{1}{2}y$. Use this information to find y explicitly.

10. (5 pt) Use a linear approximation to approximate $\sqrt{119}$. Is this approximation too big or too small? Explain.