1. (24 pt) Evaluate the following limits.

a) $\lim_{x \to 0} \cos(x^2 \sin(\frac{1}{x}))$ b) $\lim_{x \to 2} e^{\tan(\frac{x^2-4}{x^4-16})}$ c) $\lim_{x \to 0} \int_x^{2x} \frac{1}{t} dt$ d) $\lim_{x \to -\infty} \frac{\sqrt{x^2+3x}}{3x+4}$

2. (24 pt) Find the derivative of each of the following functions.

a) $f(x) = x^{x^2 e^x \sin(x)}$ b) $g(x) = F(G(x \tan(x)))$ c) $h(x) = \frac{\sec^2(x)}{\tan^2(x) + 1}$

d) $k(x) = \frac{(2x)^x}{(3x)^{2x} + 4}$

3. (24 pt) Evaluate the following integrals.

a) $\int \frac{\sec^2(x)}{\tan^2(x) + 1} dx$ b) $\int_0^{\ln(2)} \frac{e^{2x}}{e^x + 1} dx$ c) $\int_0^{\sqrt{3}} \frac{x^3}{\sqrt{x^2+1}} dx$ b) $\int \frac{dx}{x \ln(x)}$

4. (5 pt) Show that among all rectangles of some fixed perimeter P, a square has maximum area.

5. (5 pt) Use the definition of the derivative to find the derivative of the function $f(x) = \frac{1}{x} + \sqrt{x}$.

6. (5 pt) Two cars take off at 1pm from the same point. One of the cars goes due east at 50 miles per hour and the other goes northeast at 60 miles per hour. How fast is the distance between the two cars changing at 3pm?

7. (8 pt) Graph the function $f(x) = \ln(|\frac{x}{x^2+1}|)$. The first two derivatives of this function are $f'(x) = \frac{1-x^2}{x(x^2+1)}$ and $f''(x) = \frac{x^4-4x^2-1}{x^2(x^2+1)^2}$ (for your convenience the real roots of $x^4 - 4x^2 - 1$ are approximately -2.058 and 2.058).

8. (5 pt) Find the maximum and minimum values of the function $f(x) = x^{\frac{1}{3}}(1 - 2x)$ on the interval $[0, 2]$.

9. (5 pt) Show that the function

$$f(x) = x \int_1^{x^2+2} \ln(t) dt$$

has at most one root.

10. (5 pt) Find the tangent line to the curve defined by the equation

$$xe^y = ye^{xy}$$

at $(0, 0)$.