MATH 165
FALL 2009
FINAL EXAM

1. (25 pt) Evaluate the following limits if they exist.

$$\text{a) } \lim_{x\to -2} \frac{x^4-4x-24}{x^3+x^2-x+2} \qquad \text{b) } \lim_{x\to\infty} x\sin(\frac{1}{x}) \qquad \text{c) } \lim_{x\to 0}(\frac{1}{ax+1})^{(\frac{1}{bx})}$$

$$\text{d) } \lim_{x\to\infty} \frac{x+\sin(x)}{x\sqrt{x}} \qquad \text{e) } \lim_{x\to\infty} \frac{2x^3}{\sqrt{x^6+1}}$$

2. (24 pt) Find the derivatives of the following functions.

$$\text{a) } f(x) = g(\sqrt{x})h(x^2) \qquad \text{b)} f(x) = x^{(ax)^{(bx)}}$$

$$\text{c) } h(x) = \tan^{-1}(\sin(x^2+1)) \qquad \text{d) } k(x) = \int_{e^{2x}}^{x^3} \cos(t^2)dt$$

3. (18 pt) Evaluate the following integrals.

$$\text{a) } \int_{-1}^{1} \frac{\tan^{-1}(x)\tan^2(x^2)}{x^4+1}\,dx \qquad \text{b) } \int \frac{\sin(2x)}{\sin^2(x)+a^2}dx, a \neq 0 \qquad \text{c) } \int_0^{\ln(2)} \frac{e^x}{2e^x+4}dx$$

4. (5 pt) Use the definition of the derivative to find the derivative of $f(x) = \sqrt{x^2+4x}$.

5. (5 pt) Use the definition of the definite integral to find $\int_0^4 (4x-2)dx$.

6. (8 pt) Find the area of the largest triangle that can be inscribed inside a circle of radius R.

7. (5 pt) A conical tank (with circular base on top) is being filled with water at a constant rate. The tank has base radius 6 feet and height 8 feet. If the water level is rising at 1 inch per second when the water is 4 feet deep, at what rate is the tank being filled (in cubic feet per second)?

8. (10 pt) Sketch the graph of the curve

$$f(x) = \ln|x^{\frac{1}{3}}+1|.$$

For your convenience, the first two derivatives are given by

$$f'(x) = \frac{1}{3x^{\frac{2}{3}}(x^{\frac{1}{3}}+1)}$$

and

$$f''(x) = \frac{-(3x^{\frac{1}{3}}+2)}{9x^{\frac{5}{3}}(x^{\frac{1}{3}}+1)^2}.$$

9. (5 pt) A curve is defined implicitly by the equation $y^3 - x^3 = xy + 1$. Find the tangent line to this curve at the point $(0,1)$.

10. (5 pt) A spherical asteroid has a diameter of 100 miles ($\pm\frac{1}{2}$ miles). Use a linear approximation to estimate the maximum error in using the 100 mile measurement to compute the surface area of the asteroid. What is the relative error? (The surface area of a sphere of radius R is given by $S = 4\pi R^2$.)