MATH 165
FALL 2010
FINAL EXAM

1. (24 pt) Evaluate the following limits (in part c), $f(t)$ is continuous).

a) $\displaystyle\lim_{x\to 1}\frac{\ln(x)-5x+5}{x^2-1}$ b) $\displaystyle\lim_{x\to 0} x^2\tan^{-1}(\sin(\frac{1}{x}))$ c) $\displaystyle\lim_{h\to 0}\frac{\int_a^{x+h} f(t)dt-\int_a^x f(t)dt}{h}$

d) $\displaystyle\lim_{x\to -\infty}\frac{\sqrt{x^4-x}}{5x^2+4}$

2. (24 pt) Find the derivative of each of the following functions.

a)$f(x)=x^2e^{(x^2+x)}\sin(x^3)$ b)$g(x)=\dfrac{F(G(x))}{x^{x^2}+\cos(x)}$ c)$h(x)=\displaystyle\int_{x^2}^{x^3} e^{t^2}\,dt$

d)$k(x)=(\tan(x))^{(\sin(x^2))}$

3. (24 pt) Evaluate the following integrals.

a) $\displaystyle\int\frac{\ln(x^5)}{x}dx$ b) $\displaystyle\int_0^{\frac{\pi}{4}}\frac{4\sin(x)}{\cos^2(x)+1}dx$ c) $\displaystyle\int_0^1\frac{x^2+1}{x+1}dx$ d) $\displaystyle\int\frac{\cos(\sqrt{x})}{2\sqrt{x}}dx$

4. (5 pt) Find the volume of the largest cylinder that can be inscribed inside a sphere of radius R.

5. (5 pt) Use the definition of the derivative to find the derivative of the function $f(x)=\sqrt{1-7x}$.

6. (5 pt) An unstable cone of sand in the shape of a cone and volume 3000π cubic feet begins to collapse in such a way that its height is decreasing at 2 feet per second. If we assume that during the collapse, the pile always remains a cone, how fast is the radius increasing when the pile of sand is 10 feet tall?

7. (8 pt) Graph the function $f(x)=2x^{\frac{1}{3}}+x^{\frac{2}{3}}$. The first two derivatives of $f(x)$ are given by $f'(x)=\frac{2(1+x^{\frac{1}{3}})}{3x^{\frac{2}{3}}}$ and $f''(x)=\frac{-2(2+x^{\frac{1}{3}})}{9x^{\frac{5}{3}}}$.

8. (5 pt) Find the maximum and minimum values of the function $f(x)=x^{\frac{1}{3}}(7-x^2)$ on the interval $[0,2]$.

9. (5 pt) Use a linear approximation to estimate $\sqrt[3]{62}$. Is your estimate too large or too small?

10. (5 pt) Consider the circle $x^2+y^2=R^2$. Use implicit differentiation to show that the tangent line to this circle at any point is perpendicular to the radius of the circle at that point.