MATH 165
FALL 2011
FINAL EXAM

1. (24 pt) Evaluate the following limits (in part c), $0 \leq f(x) \leq |\sin(x)|$ for all x).

$$\text{a) } \lim_{x \to 0} \frac{e^{2x^2} - 1 - 2x^2}{x^4} \qquad \text{b) } \lim_{x \to 0} \frac{\sin(5\tan(4x))}{\sin(3x)} \qquad \text{c) } \lim_{x \to 1} \ln(x) f\left(\frac{1}{x^4 - 1}\right)$$

$$\text{d) } \lim_{x \to \infty} \left(\sqrt{x^2 + 1} - \sqrt{x^2 - x}\right)$$

2. (24 pt) Find the derivative of each of the following functions.

$$\text{a)} f(x) = \sin(e^{x^3}) \tan(xe^{\sin(2x)}) \qquad \text{b)} g(x) = (x\ln(x))^{\sqrt{x^2+1}} \qquad \text{c)} h(x) = \frac{F(\ln(|\sin(x)|))}{f(x)^{g(x)} + g(x)^{f(x)}}$$

$$\text{d)} k(x) = \int_{2x}^{x^2} \ln(\sinh^{-1}(t)) dt$$

3. (24 pt) Evaluate the following integrals.

$$\text{a) } \int 3x^2 \sin(x^3 + 1) dx \qquad \text{b) } \int \frac{1}{x((\ln(x))^2 + 1)} dx \qquad \text{c) } \int_0^2 x^3 \sqrt[3]{x^2 + 2} dx$$

$$\text{d) } \int_{\ln(3)}^{\ln(8)} \frac{e^x}{\sqrt{e^x + 1}} dx$$

4. (5 pt) Show that among all rectangles of fixed area A, a square has the minimal perimeter.

5. (5 pt) Use the definition of the derivative to find the derivative of the function $f(x) = \frac{2}{3x+4}$.

6. (5 pt) Two ships are at the same place on the ocean at noon. Right at noon, one of the ships heads east at 20 miles per hour and the other heads 30 degrees west of north at 15 miles per hour. How fast is the distance between the ships changing at 4pm?

7. (8 pt) Graph the function

$$\int_0^x \tan^{-1}(t) dt.$$

8. (5 pt) Find the maximum and minimum values of the function $f(x) = x^n \ln(x)$ on the interval $[\frac{1}{e^2}, e^2]$. You are free to use the results of problem 9.

9. (5 pt) Show by induction that for all integers $n \geq 1$, $2n \leq e^{2n-1}$ (and hence $2en \leq e^{2n}$).

10. (5 pt) Use the definition of the definite integral to compute $\int_0^8 (6x - 2) dx$.