MATH 166
SPRING 2003
EXAM 1

1. (48 pt) Evaluate the following integrals:

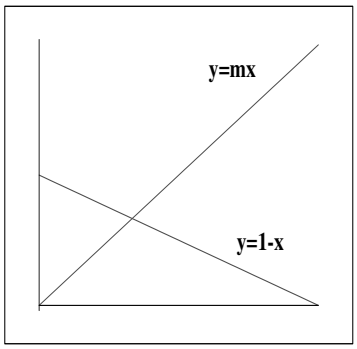
a) $\int \frac{\ln(x)}{x^2}dx$ b) $\int \frac{dx}{\sqrt{x^2+a^2}}$ $(a \neq 0)$ c) $\int_0^3 \frac{x^2dx}{(25-x^2)^{\frac{3}{2}}}$

d) $\int \frac{2x^3+6x^2+6x+2}{x^4+2x^3+2x^2}dx$ e) $\int \sin(\sqrt{x})dx$ f) $\int_0^{\frac{R}{2}} \frac{xdx}{\sqrt{R^2-x^2}}$

2. (10 pt) Suppose that 100 foot pounds of work is required to stretch a spring from its natural length of 1 foot to a length of 1.5 feet. If a 100 pound weight is hung on this spring, what will its stretched length be?

3. (16 pt) Consider the curves $f(x) = x^3 - 4x$ and $g(x) = 23x - 2x^3$.
   a) Find the area between the curves.
   b) Find the volume obtained when the part of the region bounded by the curves to the right of the y-axis is revolved about the line $x = -1$.

4. (16 pt) Consider the region bounded by the x-axis, the y-axis and the line $y = 1 - x$.
   a) Find the volume obtained when this region is revolved about the y-axis.
   b) Let $y = mx$ be a line through the origin with positive slope $(m > 0)$ and consider the region bounded by $y = mx$, $y = 1 - x$ and the x-axis. Find the value of m so that when this region is revolved about the y-axis, the resulting volume is half the volume from part a).



5. (10 pt) Let $f(x)$ be a differentiable function such that:

$$\int_0^2 x^2 f'(x)dx = 4 \text{ and } \int_0^2 xf(x)dx = 6$$

find $f(2)$.

6. (10 pt) Consider the region bounded by the curves $y = \frac{1}{1+x^2}$, the y-axis, the x-axis, and $x = n$ $(n > 0)$. Find the volume obtained when this region is revolved about the x-axis. What happens to your answer as $n \longrightarrow \infty$?