MATH 166
SPRING 2005
EXAM 1

1. (40 pt) Evaluate the following integrals:

a) $\int e^x \sqrt{e^{2x}-1}\, dx$ b) $\int \frac{3d\theta}{4+5\sin(\theta)}$ c) $\int_{-r}^{r} \frac{r^2}{(x^2+r^2)^{\frac{3}{2}}} dx$

d) $\int_0^{\frac{\pi}{4}} 2x\tan^{-1}(x)dx$ e) $\int e^{2x}\sin(x)dx$

2. (8 pt) Find the partial fraction decomposition of the following function and solve for the constants.

$$f(x) = \frac{x^4+4x^3+3x^2-2x-2}{x^4+2x^3+2x^2}$$

3. A solid object is built so that its cross sections that are parallel to its base are always in the shape of a right triangle. One of the perpendicular sides of the triangle is of length $\frac{t}{t^4+1}$ at t units above the base. The other perpendicular side is always 4 units long.

a) (9 pt) Find the volume of this object if it is h units tall.
b) (3 pt) What happens to your answer as $h \longrightarrow \infty$?

4. (16 pt) Let $a > 0$ and consider the region bounded by the lines $y = a, x = a$, the $x-$axis, and $y = mx$ $(m \geq 1)$.

a) Find the volume obtained when this region is revolved about the $x-$axis. What happens to your answer as $m \longrightarrow \infty$?
b) Find the volume obtained when this region is revolved about the line $x = -a$. What happens to your answer as $m \longrightarrow \infty$?

5. A tank is in the shape of an inverted cone. The base radius is R and the height is $2R$ (all measured in feet).

a) (10 pt) If this tank is filled with water, how much work is required to pump all of the water out of the tank (water weighs 62.5 pounds per cubic foot)?
b) (4 pt) If the tank is turned upside down (placed with the circular base on the ground), will it take more or less work to pump the water out (explain your answer)?

6. (10 pt) Let $f(x)$ be a positive continuous increasing function such that the area under the curve from $x = 0$ to $x = 4$ is 4. Additionally the volume obtained by revolving the region bounded by $x = 0, x = 4, y = 0$, and $y = f'(x)$ about the $y-$axis is 8π. Compute $f(4)$.

7. (10 pt) Suppose that the average value of $f(x)$ is -9 on the interval $[2, 4]$ and 7 on the interval $[-4, 2]$. What is the average value of the function on the interval $[-4, 4]$? Can $f(x)$ be an odd function?