MATH 166
SPRING 2006
EXAM 1

1. (40 pt) Evaluate the following integrals:

$$\text{a)} \int \frac{x^2}{\sqrt{4x^2-8x}}\,dx \qquad \text{b)} \int \frac{dx}{e^x+e^{2x}} \qquad \text{c)} \int_{-r}^{r} \frac{(r^2-x^2)^{\frac{3}{2}}}{r^4}\,dx$$

$$\text{d)} \int_1^e x^n \ln(x^m)dx;\ n,m \geq 0 \qquad \text{e)} \int e^{-x}\sin(2x)dx$$

2. (20 pt) Consider the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1;\ a,b>0$.
   a) Find the area enclosed by this ellipse.
   b) Find the volume obtained when the right half of this ellipse is revolved about the $y-$axis.

3. (10 pt) Two springs with spring constants k_1 and k_2 respectively are attached. If this "double spring" is stretched L units beyond its natural length, find how much the first spring (the one with spring constant k_1) is stretched (hint: say that the first spring is stretched a units and minimize the work done).

4. (15 pt) A sphere of radius 1 has a volume of $\frac{4}{3}\pi$. You wish to make a "napkin ring" out of this by drilling a hole of radius r all the way through the sphere. How big should r be so that the volume of the resulting "napkin ring" is exactly π?

5. (15 pt) A tank is in the shape of an inverted circular cone and is full of water. The base radius of the cone is R and the height is h. Additionally, a spigot it built on the "side" of the conical tank at height d. How high do we have to place the spigot such that the work done in pumping the cone dry is exactly 0 (in other words, find d in terms of R and h so that the total work done in emptying the tank is 0)?

6. (10 pt) Consider the function $f(t)=4t-t^2$. Find the interval $[a,b]$ of length 2 where the average value of the function on $[a,b]$ is maximal. What is this maximal average value?