1. (40 pt) Evaluate the following integrals:

   a) \[ \int \frac{x^2}{\sqrt{4x^2 - 8x}} \, dx \]
   b) \[ \int \frac{dx}{e^x + e^{2x}} \]
   c) \[ \int_{-r}^{r} \frac{(r^2 - x^2)^{\frac{3}{2}}}{r^4} \, dx \]
   d) \[ \int_{1}^{e} x^n \ln(x^m) \, dx; \quad n, m \geq 0 \]
   e) \[ \int e^{-x} \sin(2x) \, dx \]

2. (20 pt) Consider the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; \quad a, b > 0 \).
   a) Find the area enclosed by this ellipse.
   b) Find the volume obtained when the right half of this ellipse is revolved about the \( y \)-axis.

3. (10 pt) Two springs with spring constants \( k_1 \) and \( k_2 \) respectively are attached. If this “double spring” is stretched \( L \) units beyond its natural length, find how much the first spring (the one with spring constant \( k_1 \)) is stretched (hint: say that the first spring is stretched \( a \) units and minimize the work done).

4. (15 pt) A sphere of radius 1 has a volume of \( \frac{4}{3} \pi \). You wish to make a “napkin ring” out of this by drilling a hole of radius \( r \) all the way through the sphere. How big should \( r \) be so that the volume of the resulting “napkin ring” is exactly \( \pi \)?

5. (15 pt) A tank is in the shape of an inverted circular cone and is full of water. The base radius of the cone is \( R \) and the height is \( h \). Additionally, a spigot it built on the “side” of the conical tank at height \( d \). How high do we have to place the spigot such that the work done in pumping the cone dry is exactly 0 (in other words, find \( d \) in terms of \( R \) and \( h \) so that the total work done in emptying the tank is 0)?

6. (10 pt) Consider the function \( f(t) = 4t - t^2 \). Find the interval \([a, b]\) of length 2 where the average value of the function on \([a, b]\) is maximal. What is this maximal average value?