MATH 166
SPRING 2008
EXAM 1

1. (50 pt) Evaluate the following integrals:

a) $\int \sin(x)\sqrt{\cos^2(x)+4}\,dx$ b) $\int e^x \sin(x)\cos(x)\,dx$ c) $\int \frac{x^6+3x^4+x^3+2x^2-1}{x^6+2x^4+x^2}dx$

d) $\int_0^{\frac{1}{2}} \sin^{-1}(x)dx$ e) $\int_1^3 \frac{1}{4x-x^2}dx$

2. (8 pt) The amount of force exerted on a rocket ship of mass m on a planet of mass M is given by the formula

$$F = \frac{GMm}{R^2}$$

where G is the gravitational constant of the universe and R is the distance from the center of the planet. If the radius of the planet is R_0, find the work required for the rocket to "blast" to a distance, D from the center of the planet. What happens to your answer as $D \longrightarrow \infty$?

3. (10 pt) Let $y = f(x)$ be a positive function, $a \geq 0$ be a constant, and let $\mathfrak{R}$ be the region bounded by $f(x)$, $x = t$ $(t > 0)$, the $x-$axis, and the $y-$axis. Suppose that for every positive value of t, the volume obtained when this region $\mathfrak{R}$ is revolved about the $x-$axis is precisely the same as the volume obtained when $\mathfrak{R}$ is revolved about the line $x = -a$. Find $f(x)$.

4. (24 pt) Consider the quarter of the circle $x^2 + y^2 = R^2$ $(R > 0)$ that lies in the first quadrant and the line $y = mx$, with $m \geq 0$.

a) Find the volume obtained when the region in the first quadrant bounded by the $x-$axis, the circle, and $y = mx$ is revolved about the $y-$axis.
b) What happens to your answer from a) as $m \longrightarrow \infty$ (and does this make sense)?
c) Find the volume obtained when the region in the first quadrant bounded by the $x-$axis, the circle, and $y = mx$ is revolved about the x-axis.
d) What happens to your answer from c) as $m \longrightarrow \infty$ (and does this make sense)?

5. (10 pt) A trough L units long is placed on the ground. The ends of the trough look like trapezoid with the parallel sides (bottom and top respectively) of lengths a and b. The height of the trapezoid is h. If the trough is filled with a liquid of density ρ, how much work is required to pump all of the fluid out of a spigot that is k units above the top of the trough?

6. (8 pt) Suppose that $f(x)$ has $y = c$ as a horizontal asymptote to the right ($\lim_{x\to\infty} f(x) = c$). Let $A(x)$ denote the average value of the function $f(x)$ on the interval $[a, x]$ (where a is constant). Find $\lim_{x\to\infty} A(x)$.