

MATH 166
SPRING 2010
EXAM 1

1. (50 pt) Evaluate the following integrals:

$$\begin{array}{lll} \text{a) } \int_0^1 \frac{x^3}{(4-3x^2)^{\frac{3}{2}}} dx & \text{b) } \int \cos(\ln(x)) dx & \text{c) } \int \frac{1}{\sqrt{x^2+2Rx+2R^2}} dx, R > 0 \\ \text{d) } \int \frac{d\theta}{3\cos(\theta)+4\sin(\theta)} & \text{e) } \int_0^{\ln(2)} (x^2+x)e^x dx & \end{array}$$

2. (15 pt) Find the volume obtained when the region bounded by the x -axis and $f(x) = ax(1-x)$ is revolved about the line $x = 3$.

3. (12 pt) An hourglass is made by revolving $f(x) = |a \sin(x)|, 0 \leq x \leq 2\pi$ about the x -axis. Find the value of a so that the upper chamber of the hourglass can hold 2π cubic units of sand.

4. (10 pt) A wedge is created by slicing a half cylinder of radius R with a plane at angle θ ($0 \leq \theta < \frac{\pi}{2}$). Find the volume of this object.

5. (15 pt) We define the function

$$g(x) = \frac{1}{x-a} \int_a^x f(t) dt, x > a,$$

where $f(x)$ is continuous.

- a) Show that if $g(x)$ has a critical number then it occurs at a point where $f(x)$ is equal to its mean value on $[a, x]$.
- b) Evaluate $\lim_{x \rightarrow a} g(x)$.
- c) Suppose that $f(x)$ has $y = m$ as a horizontal asymptote. Evaluate $\lim_{x \rightarrow \infty} g(x)$ (you may assume that $\lim_{x \rightarrow \infty} \int_a^x f(x) dx = \infty$).

6. (8 pt) A water storage tank is in the shape of a circular cylinder of radius R and height h (both measured in feet). This tank is buried on its side (horizontal position) D feet below the ground. If water weighs ρ pounds per cubic foot find the work done in pumping the water out of the tank through a spigot that is a feet above the ground.