MATH 166
SPRING 2013
EXAM 1

1. (50 pt) Evaluate the following integrals:

a) $\int_0^9 e^{\sqrt{x}} dx$ b) $\int \frac{16}{(x^2+2x+5)^2} dx$ c) $\int_{4a}^{5a} \sqrt{x^2-16a^2}\, dx$

d) $\int \frac{8x^2}{x^4-16}\, dx$ e) $\int \sin(2\ln(x)) dx$

2. (10 pt) An object has volume given by the formula $V(h) = (a^2 + ah + h^2)h$ where $a > 0$ is a fixed number and h is the height of the object (for any height h). Find the area of the cross section of this object at height a.

3. (20 pt) Consider the region bounded by the functions $y = \cos(x)$, $y = \sin(x)$ and the $y-$axis. Let V be the volume obtained when this region is revolved about the $x-$axis and W be the volume obtained when this region is revolved about the line $x = -a$ where $a \geq 0$. Find the value of a so that these two volumes are the same.

4. (10 pt) Suppose that you have a spring. You hang a weight of F pounds from the spring and this stretches the spring a feet. Show that the amount of work done in this stretch is $\frac{1}{2}Fa$.

5. (10 pt) Let $g(x)$ be a differentiable function. If $g(x)$ represents the average value of the function $f(t)$ on the interval $[a, x]$, find $f(x)$.

6. Consider the region bounded by the $y-$axis and the curve $x = R - \frac{R}{h^2}y^2$.
   a) (6 pt) Find the volume obtained when this region is revolved about the $y-$axis.
   b) (4 pt) If this object is raised on a column a feet high, how much work is required to pump it full of a liquid of density ρ?