

**MATH 166**  
**SUMMER 2011**  
**EXAM 1**

1. (32 pt) Evaluate the following integrals.

a)  $\int \tan(x) dx$     b)  $\int \frac{e^x}{e^{2x} + 1} dx$     c)  $\int_0^1 8x^3 \sqrt{2x^2 + 3} dx$   
d)  $\int \frac{1}{x^{\frac{1}{2}} - x^{\frac{1}{3}}} dx$

2. (20 pt) Consider a truncated pyramid with square bases of lengths  $a$  and  $b$  respectively and height  $h$ .

- a) Find the volume of this pyramid.
- b) What (should) happens to your formula in the case where  $a = b$ ?  $a = 0$ ?  $b = 0$ ?

3. (24 pt) Consider the region bounded by the functions  $f(x) = x$  and  $g(x) = x^2$ .

- a) Find the volume obtained when this region is revolved about the  $x$ -axis.
- b) Find the volume obtained when this region is revolved about the line  $x = -3$ .

4. (12 pt) Consider the region in the first quadrant bounded by the curve  $f(x) = 3x - x^3$  and the  $x$ -axis. Find the volume obtained when this region is revolved about the line  $x = -a$ ,  $a > 0$ .

5. (12 pt) Find the work required to pump a liquid of density  $\rho$  out of a large tank shaped like a half cylinder (lying on its side) of radius  $R$  and length  $L$ .

6. (10 pt) Let  $f(x)$  be a continuous function with the property that the average value of  $f(x)$  on the interval  $[0, x]$  is equal to  $\sin(x)$ . Find  $f(x)$ .