MATH 166
SPRING 2003
EXAM 2

1. (32 pt) Evaluate the following integrals:

$$\text{a)} \int_0^\infty \frac{\cos(x)}{1+\sin^2(x)}dx \qquad \text{b)} \int_{-\infty}^{-1} \frac{dx}{\sqrt{x^2+1}} \qquad \text{c)} \int_{-\infty}^{\infty} e^{-|x|}dx$$

$$\text{d)} \int_{-\infty}^{\infty} f(x)dx \text{ where } f(x) = \begin{cases} |\frac{1}{\sqrt[3]{x}}|, & \text{if } 0 < |x| \leq 1; \\ \frac{2}{x^2+1}, & \text{if } 1 < |x|. \end{cases}$$

2. (8 pt) Suppose that you have a function $f(x)$. Let $g(x) = \int_0^x f(t)dt$ and let $F(x)$ be an antiderivative of $g(x)$. and you know that $-6 < f^{(4)}(x) \leq 4$ for all $1 \leq x \leq 5$ and $-3 \leq f''(x) \leq 2$. Find the smallest value of n for which the error of Simpson's rule used in estimating $\int_1^5 F(x)dx$ is less than $\frac{1}{81}$.

3. (20 pt) Consider the region between the curves $f(x) = 4x$ and $g(x) = x^2$.
   a) Find the length of the boundary of this region.
   b) Find the surface area of the solid generated when this region is revolved about the y-axis.

4. (24 pt) Consider the region bounded by the function $f(x) = \sin(ax)$, $0 \leq x \leq \frac{\pi}{a}$ and the x-axis.
   a) Find the centroid of this region.
   b) Find the volume obtained when this region is revolved about the x-axis.
   c) Find the volume obtained when this region is revolved about the y-axis.
   d) For what value(s) of a do the two volumes coincide?

5. (10 pt) Find the surface area generated when the curve $f(x) = e^{-x}$, $x \geq 0$ is revolved about the x-axis.

6. (8 pt) Find the force due to hydrostatic pressure on the side of a glass aquarium if the side is rectangular of width w and height h. You may assume that the aquarium is filled to the top and that water weighs 62.5 pounds per cubic foot.

7. (8 pt) Find the volume of a "hollowed torus" that is obtained from revolving an annulus of inner radius r and outer radius R about an axis (assume that this axis is a units from the center of the annulus where $a \geq R$). Use your result to obtain the volume of a solid torus. Then obtain the surface area of the solid torus.