MATH 166
SPRING 2004
EXAM 2

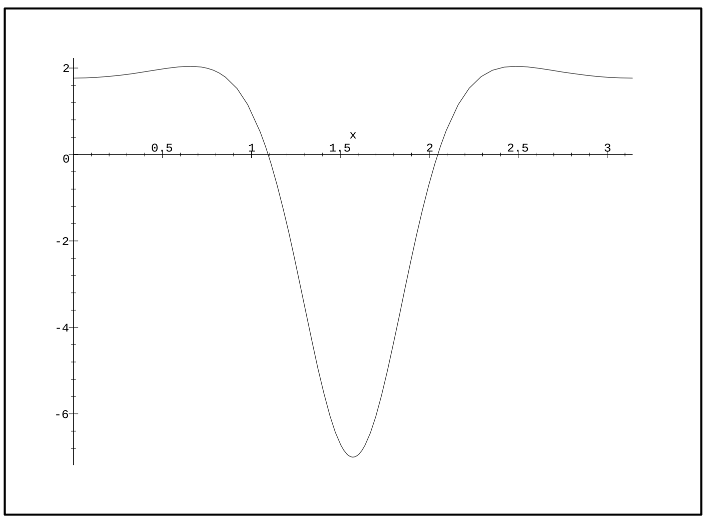
1. (32 pt) Evaluate the following integrals if they exist:

$$\text{a)} \int_0^\infty \frac{dx}{x^{\frac{4}{3}} + x^{\frac{2}{3}}} \qquad \text{b)} \int_1^\infty 4x^3 e^{-x^2} dx \qquad \text{c)} \int_{-\infty}^\infty \frac{2\,dx}{x^2-1} \qquad \text{d)} \int_e^\infty \frac{dt}{t\ln(t)}$$

2. (12 pt) A truck is hauling a (circular) cylindrical tank filled with a fluid that weighs a pounds per cubic foot. If the circular end of the tank is of radius R and the length of the tank is L (both measured in feet), find the force due to hydrostatic pressure on the circular end of the tank.

3. (20 pt) Consider the region bounded by the curves $f(x) = \sin(ax), 0 \le x \le \frac{\pi}{a}$ and the $x-$axis.
   a) Find the surface area of the solid generated when this region is revolved about the $x-$axis.
   b) Find the integral (do not attempt to evaluate) that expresses the length of the curve $f(x) = \sin(ax), 0 \le x \le \frac{\pi}{a}$.

4. (10 pt) Suppose that you want to use Simpson's rule to estimate $\int_0^3 \sqrt{1+\cos^2(x)}\,dx$. Find the smallest value of n so that using S_n to estimate this integral will produce an error of no more than $\frac{7}{600{,}000}$. The fourth derivative of $f(x) = \sqrt{1+\cos^2(x)}$ is pictured below.



5. (24 pt) Consider the function(s) $f(x) = \frac{1}{x^n}$, with $n > 2$, and the region bounded by this function for $x \ge 1$ and the $x-$axis.
   a) Compute the area of this region.
   b) Find the volume of the solid obtained when this region is revolved about the $x-$axis and compute the $y-$coordinate of the centroid of the region.
   c) Find the volume of the solid obtained when this region is revolved about the $y-$axis and compute the $x-$coordinate of the centroid of the region.

6. (12 pt) Suppose that you have a region $\mathfrak{R}$ of area A completely in the first quadrant of the plane. Suppose that the centroid of this region is located at the point (a, b). What is the slope of the line through the origin (i.e. a line of the form $y = mx$ with $m < 0$) that you should revolve the region $\mathfrak{R}$ about to give an object of maximal volume? Explain, and find this maximal volume.