

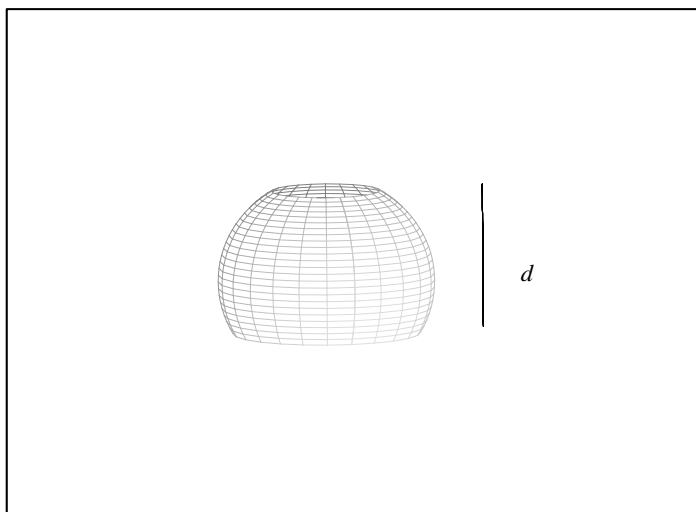
MATH 166
SPRING 2006
EXAM 2

1. (32 pt) Evaluate the following integrals if they exist:

a) $\int_0^{\infty} \frac{2f'(x)}{(f(x))^2 + 1} dx$, where $\lim_{x \rightarrow \infty} f(x) = \infty$ b) $\int_{-\infty}^{\infty} e^{\sqrt{|x|}} dx$ c) $\int_{-1}^1 \frac{x}{\sqrt{1-x^4}} dx$ d) $\int_0^e t^n \ln(t) dt$

2. (16 pt) Consider the function $f(x) = \int_{\frac{\pi}{4}}^x \sqrt{\tan^2(t) - 1} dt$ with $\frac{\pi}{4} \leq x < \frac{\pi}{2}$. Find the length of the curve $y = f(x)$ where $\frac{\pi}{4} \leq x \leq \frac{\pi}{3}$.

3. (16 pt) Consider a sphere of radius R . Find the surface area of the part of the sphere pictured (the bottom and top of the sphere are sliced by parallel planes so that the resulting piece of the sphere stands at height d). In computing the surface area, you can ignore the circular “poles”.



4. You wish to approximate the value of

$$\int_0^1 e^{-x^2} dx$$

- a) (5 pt) Find the value of K if you wish to use the trapezoid rule, and find the value of K if you wish to use the midpoint rule.
- b) (5 pt) Find n such that the error from using T_n is less than or equal to $\frac{1}{600}$.
- c) (5 pt) Find n such that the error from using M_n is less than or equal to $\frac{1}{600}$.

For your convenience, if $f(x) = e^{-x^2}$ then $f''(x) = 2(2x^2 - 1)e^{-x^2}$ and $f^{(4)}(x) = 4(4x^4 - 12x^2 + 3)e^{-x^2}$.

5. (16 pt) For what value(s) of m is the centroid of the region bounded by $y = mx$, the x -axis, the y -axis, and the line $x = b$ (where b is some positive constant) located on the line $y = x$? For this value of m , calculate the volume of the solid obtained by revolving this region about the line $y = -x$.

6. (15 pt) An undersea house has glass roof that is at an angle of $\frac{\pi}{6}$ radians (with respect to the horizontal sea floor). If the roof is L feet long, w feet wide, and the top of the roof is at a depth D beneath the sea, find the force due to hydrostatic pressure on the roof (for this system of units, pressure is equal to 62.5 times the depth).