# MATH 166
# SPRING 2007
# EXAM 2

1. (30 pt) Evaluate the following integrals if they exist.

$$\text{a)}\int_0^\infty \frac{dx}{\sqrt{x}(x+1)} \qquad \text{b)}\int_0^1 \frac{dx}{x(\ln(x))^2} \qquad \text{c)}\int_{-\infty}^{-2} \frac{2}{x^2-1}\,dx$$

2. Let $k$ be a constant.

a) (6 pt) Show that the integral $\int_{-\infty}^{\infty} x dx$ diverges.

b) (6 pt) Find a function, $f(t)$, satisfying

$$\lim_{t\to\infty} f(t) = \infty$$

and

$$\lim_{t\to\infty} \int_{-t}^{f(t)} x dx = k.$$

3. Let $f(x)$ be a non-decreasing function with a continuous derivative.

a) (6 pt) Show that $\int_a^b \sqrt{1+(f'(x))^2}dx \leq \int_a^b \sqrt{1+2f'(x)+(f'(x))^2}dx$.

b) (6 pt) Use part a) to show that if $L$ is the arclength of $f(x)$ from $a$ to $b$ then

$$L \leq (b-a) + (f(b) - f(a)).$$

4. (15 pt) Find the surface area obtained when the upper half of the ellipse $\frac{x^2}{4} + y^2 = 1$ is revolved about the $x-$axis.

5. A rectangular window of width $w$ and height $h$ on an aquarium is submerged so that the top is $D$ units below the surface of the liquid. You may assume that the pressure is given by $P = kd$ where $k$ is a constant and $d$ is the depth beneath the surface.

a) (10 pt) Find a formula for the force due to hydrostatic pressure acting on the window.

b) (5 pt) If $D = 0$ and $w$ remains constant, how much do we have to increase $h$ to double the force on the window?

6. Consider the region bounded by the functions $y = mx$, $m \geq 0$ and $y = x - x^2$.

a) (6 pt) Find a formula for the area of this region.

b) (6 pt) Find the value(s) of $m$ for which the $y-$coordinate of the centroid is maximal.

c) (6 pt) For the value(s) of $m$ found in part b), find the volume obtained when the region is revolved about the $x-$axis.

7. (8 pt) Suppose that you want to approximate

$$\int_a^b (Ax^4 + Bx^3 + Cx^2 + Dx + E)dx$$

using Simpson's rule. Show that the error in using Simpson's rule is never more than $\frac{|A|}{120}L^5$, where $L$ is the length of the interval of integration.