1. (40 pt) Evaluate the following integrals if they exist:

a) \( \int_0^\infty (x - \sqrt{x^2 + 1}) \, dx \)

b) \( \int_{-\infty}^\infty e^{-\sqrt{|x|}} \, \frac{dx}{x^3} \)

c) \( \int_{-R}^R \frac{dx}{\sqrt{R^2 - x^2}} \)

d) \( \int_{-e}^{e^2} \frac{(\ln(|t|))^n dt}{t}, n > 0 \)

2. (20 pt) Let \( a > 0 \) be a fixed constant. Find all values of \( b \) \((a \geq b \geq 0)\) such that the centroid of the region bounded by \( f(x) = ax(1 - x) \) and \( g(x) = bx(1 - x), 0 \leq x \leq 1 \) is actually in the region.

3. (8 pt) Show that the substitution \( x = \tan(\theta) \) converts the integral

\( \int_1^\infty \frac{\sqrt{x^2 + 1}}{x^n} \, dx, n \) an integer greater than 2

to a proper integral (and hence the integral converges).

4. (15 pt) Find the surface area obtained when \( f(x) = \sin(nx) \), where \( n \) is a positive integer and \( 0 \leq x \leq \frac{\pi}{n} \) is revolved about the \( x\)-axis. What happens to your answer as \( n \to \infty \) (does this make sense)?

5. (8 pt) Find \( a \) such that the length of the curve \( f(x) = x^3 \) on the interval \([a, a + k]\) (where \( k > 0 \) is a fixed constant) starting at \( a \) is minimized.

6. (12 pt) Find the force due to hydrostatic pressure on the side of a trough of length \( L \) if the end is a semicircle of radius \( R \) and it is filled to the top with a fluid of density \( \rho \).

7. (7 pt) Explain the construction behind Simpson’s Rule and why the value of \( n \) must always be even.