MATH 166
SPRING 2010
EXAM 2

1. (40 pt) Evaluate the following integrals:

$$\text{a) } \int_0^{\sqrt{13}} \frac{2x}{\sqrt[3]{x^2-4}}dx \qquad \text{b) } \int_0^{\infty} \frac{\ln(|x|)}{x}dx \qquad \text{c) } \int_0^{\infty} \frac{2e^{2x}}{1+e^{4x}}dx$$

$$\text{d) } \int_{-\infty}^{\pi} \cos(x)e^{\sin(x)}dx$$

2. (15 pt) Find the length of the curve $f(x) = \int_{\frac{\pi}{4}}^{x} \sqrt{\tan^2(t)-1}dt$, $\frac{\pi}{4} \leq x \leq \frac{\pi}{3}$.

3. (15 pt) Consider the region in the upper half plane ($y \geq 0$) bounded by the semicircles $y = \sqrt{R^2 - x^2}$ and $y = \sqrt{r^2 - x^2}$ with $R > r$. Locate the centroid of this region. For what value(s) of r (in terms of R) is the centroid located on the circle $y = \sqrt{r^2 - x^2}$?

4. (10 pt) Let $n \geq 1$. Show that the surface area obtained when the function $f(x) = x^n, 0 \leq x \leq 1$ is revolved about the $x-$axis is precisely the same as when the function $f(x) = x^{\frac{1}{n}}, 0 \leq x \leq 1$ is revolved about the $y-$axis.

5. (15 pt) A submerged window is in the shape of an equilateral triangle of side length a. The window has one of the vertices pointing straight down (so it has a flat top). If the pressure is ρ times the depth and the top of the window is D feet below the surface, find the force due to hydrostatic pressure on the window.

6. (15 pt) Consider the function $f(x) = \int_0^x e^{-t^2}dt$ and let $g(x)$ be its antiderivative. Suppose that I want to integrate this function on from 0 to 2. Find the appropriate values of K for the midpoint rule and for Simpson's rule.

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(6) $\sin(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(7) $\cos(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$
(8) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(9) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(10) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(11) Force=(pressure)(area) and pressure=ρ(depth).
(12) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}d\theta$
(13) $S = \int_a^b 2\pi(x \text{ or } y)ds$
(14) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(15) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(16) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(17) $A = \int_a^b \frac{1}{2}r^2 d\theta$
(18) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(19) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$