MATH 166
SUMMER 2012
EXAM 2

1. (60 pt) Evaluate the following integrals.

a) $\displaystyle\int \frac{dx}{1-\sin(x)}$ b) $\displaystyle\int_0^{a\sqrt{3}} \frac{x^2}{\sqrt{x^2+a^2}}\,dx$ c) $\displaystyle\int \frac{2x}{x^4-16}\,dx$

d) $\displaystyle\int \frac{dx}{x^2+4x+5}$ e) $\displaystyle\int e^{2x}\cos(x)\,dx$

2. (20 pt) Evaluate the following integrals.

a) $\displaystyle\int_0^{\infty} \frac{\cos(x)}{1+\sin^2(x)}\,dx$ b) $\displaystyle\int_0^{\infty} \frac{dx}{\sqrt{x}(x+1)}$

3. (20 pt) Suppose we want to estimate $\int_0^{\sqrt{15}} f(x)dx$. Suppose that we also know that $f(x) = \frac{1}{2}f'(x)$.
   a) Show that the value of K for Simpson's rule is precisely 4 times the value of K for the trapezoid rule.
   b) Find the value of n such that the error bound for S_n is equal to the error bound for T_n.

4. (10 pt) Consider the region bounded by the function $f(x) = \sqrt{x}e^{-x}, x \geq 1$ and the $x-$axis. Find the volume obtained when this region is revolved about the $x-$axis.

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $\sin(A)\cos(B) = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
(6) $\sin(A)\sin(B) = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
(7) $\cos(A)\cos(B) = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$
(8) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(9) $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(10) $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
(11) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(12) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(13) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(14) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}d\theta$
(15) $S = \int_a^b 2\pi(x \text{ or } y)ds$
(16) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(17) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(18) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(19) $A = \int_a^b \frac{1}{2}r^2 d\theta$
(20) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(21) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$
(22) If $t = \tan(\frac{x}{2})$ then $\sin(x) = \frac{2t}{t^2+1}, \cos(x) = \frac{1-t^2}{t^2+1}, dx = \frac{2dt}{t^2+1}$