MATH 166
SPRING 2003
EXAM 3

1. (18 pt) Determine if the following sequences converge or diverge.

a) $\left\{\dfrac{\tan^{-1}(\sin(n))}{2n+1}\right\}_{n=0}^{\infty}$ b) $\{a_1, a_1+a_2, \cdots, a_1+a_2+\cdots+a_n, \cdots\}$ where $\lim\limits_{n\to\infty}\dfrac{a_{n+1}}{a_n}=\dfrac{3}{4}$

c) $\{a_n\}_{n=1}^{\infty}$ where $a_n = f(n)$ with $f'(x) < 0$ and $f(x) > 0$.

2. (36 pt) Determine if the following series converge or diverge.

a) $\displaystyle\sum_{n=1}^{\infty}\frac{\ln(n)}{\ln(n^2+1)}$ b) $\displaystyle\sum_{n=0}^{\infty}\frac{\sqrt[3]{2n^3+n}}{\sqrt[3]{n^7+n^6+3}}$ c) $\displaystyle\sum_{n=2}^{\infty}\frac{(3n^3+1)^{2n}}{(2n^2+1)^{3n}}$

d) $\displaystyle\sum_{n=0}^{\infty}\frac{(n!)^3}{(3n)!}$ e) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{1}{n!}$ f) $\displaystyle\sum_{n=0}^{\infty}\frac{n\cos(n^2)}{n^3+1}$

3. (18 pt) Suppose that the power series $\sum_{n=0}^{\infty} a_n x^{2n}$ has radius of convergence equal to nine. Find the following.

a) The radius of convergence of the series $\sum_{n=0}^{\infty} a_n x^n$.
b) The radius of convergence of the series $\sum_{n=0}^{\infty} \sqrt{a_n} x^{2n}$.
c) $\lim_{n\to\infty}(64^n a_n)$.

4. (8 pt) Find the center, radius, and interval of convergence for the power series

$$\sum_{n=1}^{\infty}\frac{n(3x-2)^n}{(n^2+1)3^{2n}}.$$

5. (8 pt) Find the Maclaurin series for the function $f(x) = \tan^{-1}(x^2)$.

6. (12 pt) Consider the function $f(x) = \cos(x^2)$.

a) Find the Maclaurin series for $f(x)$.
b) Use your result from a) to find an infinite series for $\int_0^{\frac{1}{10}} \cos(x^2)dx$.
c) How many terms from this series are necessary so that the approximation $s \approx s_n$ has error less than $\frac{1}{1{,}000{,}000}$?

7. (10 pt) It is known that $\sum_{n=0}^{\infty}(-1)^n\frac{1}{2n+1} = \frac{\pi}{4}$ and $\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$. The questions below involve using the approximation $s \approx s_n$.

a) How many terms of $\sum_{n=0}^{\infty}(-1)^n\frac{1}{2n+1}$ are needed to estimate $\frac{\pi}{4}$ with error less than or equal to $\frac{1}{100}$?
b) How many terms of $\sum_{n=1}^{\infty}\frac{1}{n^2}$ are needed to estimate $\frac{\pi^2}{6}$ with error less than or equal to $\frac{1}{100}$?