1. (36 pt) Determine if the following series converge or diverge.

   a) \( \sum_{n=1}^{\infty} \frac{n^3}{\sqrt{n^3} + 1} \)
   b) \( \sum_{n=1}^{\infty} n^2 \sin\left(\frac{1}{n^4}\right) \)
   c) \( \sum_{n=0}^{\infty} \frac{28^n(n!)^3}{(3n)!} \)
   d) \( \sum_{n=1}^{\infty} \frac{(2n^2 + 1)^{3n}}{(3n^3 - 1)^{2n}} \)
   e) \( \sum_{n=2}^{\infty} (-1)^n \frac{\ln(\ln(n))}{\ln(n)} \)
   f) \( \sum_{n=1}^{\infty} \sin(a_n) \), where \( \sum_{n=1}^{\infty} a_n \) is a convergent, positive term series.

2. (12 pt) Determine if the following sequences converge or diverge.

   a) \( \left\{ n \tan\left(\frac{1}{n}\right) \right\}_{n=1}^{\infty} \)
   b) \( \left\{ a_n \right\}_{n=1}^{\infty} \) where \( a_1 = 1 \) and \( a_{n+1} = 3 - \frac{1}{a_n} \) for \( n \geq 1 \).

3. (15 pt) Suppose that the power series \( \sum_{n=0}^{\infty} a_n x^n \) has interval of convergence \([-R, R] \) \((R > 0)\) and that the series converges absolutely at the endpoints. Find the interval of convergence of the series

   \[ \sum_{n=0}^{\infty} \frac{a_n}{n+1} (ax - b)^{cn} \]

   where \( a \neq 0 \) and \( c \) is a positive integer.

4. Let \( R > 0 \) and suppose that \( \sum_{n=0}^{\infty} a_n x^n \) converges if \( |x| < R \) and diverges if \( |x| > R \).
   a) (10 pt) Show that \( \sum_{n=0}^{\infty} a_n \left(\frac{1}{2}\right)^n \) converges for \( |x| > \frac{1}{R} \) and diverges for \( |x| < \frac{1}{R} \).
   b) (5 pt) Suppose that \( f(x) \) is equal to its Maclaurin series and has the property that \( f(x) = f\left(\frac{1}{x}\right) \) for all \( x \neq 0 \). Explain why if \( R > 1 \), then \( f(x) \) is continuous everywhere and if \( R = 1 \) then \( f(x) \) has at most two discontinuities. What happens if \( R < 1 \)?

5. (12 pt) Find the center, radius, and interval of convergence of the power series

   \[ \sum_{n=1}^{\infty} \frac{\sqrt[3]{(2n+1)(3x-6)^{2n}}}{9^n} \]

6. (10 pt) Evaluate the following limit (you might try using a series).

   \[ \lim_{x \to 0} \frac{\sin(x^2) + e^{-x^2} - \cos(x^3)}{3x^4} \]

7. (10 pt) Estimate

   \[ \int_{0}^{1} \frac{dx}{16 + x^4} \]

   with error less than or equal to \( \frac{1}{20000} \) (and explain how you come to your conclusions).