MATH 166
SPRING 2011
EXAM 3

1. (48 pt) Determine if the following series converge of diverge.

a) $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{\ln(\ln(n))}{\ln(n)}$ b) $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n}\sin(3^n)}{n^2+1}$ c) $\displaystyle\sum_{n=1}^{\infty}\frac{100^n(n!)^5}{(2n)!(3n)!}$ d) $\displaystyle\sum_{n=2}^{\infty}\ln(\frac{n}{n+2})$

e) $\displaystyle\sum_{n=3}^{\infty}\frac{1}{n^p-n^q}$, where $p>q>0$. f) $\displaystyle\sum_{n=2}^{\infty}(\alpha^{\frac{1}{n^2}}-\beta^{\frac{1}{n^2}}), \alpha>\beta>0$

2. (16 pt) Consider the following sequences.

a) Show that the sequence defined by $a_{n+1}=3-\frac{3}{a_n}, n\geq 1$ diverges no matter what the value of a_1 is.
b) Show that the sequence $\{\sqrt{3}, \sqrt{3+\sqrt{3}}, \sqrt{3+\sqrt{3+\sqrt{3}}}, \cdots\}$ converges.

3. (16 pt) Consider the series $\sum_{n=2}^{\infty}\frac{1}{n\ln(n)}$ and $\sum_{n=2}^{\infty}(-1)^n\frac{1}{\ln(n)}$.

a) Explain why the series $\sum_{n=2}^{\infty}\frac{1}{n\ln(n)}$ diverges. Estimate how many terms are needed for the partial sum to exceed 100.
b) Explain why the sum $\sum_{n=2}^{\infty}(-1)^n\frac{1}{\ln(n)}$ converges. How many terms are necessary to ensure that the partial sum is within $\frac{1}{100}$ of the sum of the series.

4. (12 pt) Find the center, radius, and interval of convergence of the power series

$$\sum_{n=2}^{\infty}\frac{(2x-1)^{3n}}{64^n\ln(n)}.$$

5. (10 pt) Use a Maclaurin series to estimate

$$\int_0^1 x^2\sin(x^2)dx$$

with error no more than $\frac{1}{1000}$.

6. (8 pt) Suppose that $\sum_{n=1}^{\infty}a_n$ is a positive term series that converges by the root test (that is $\lim_{n\to\infty}(a_n)^{\frac{1}{n}}=L<1$). Let $f(x)$ be a positive function such that $\lim_{n\to\infty}f(x)=k>0$ (we will even allow $k=\infty$).

a) Show that $\sum_{n=1}^{\infty}(a_n)^{f(n)}$ converges.
b) If we only know that $\sum_{n=1}^{\infty}(a_n)^{f(n)}$ converges, does this mean that $\sum_{n=1}^{\infty}a_n$ converges?

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(6) $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(7) $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
(8) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(9) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(10) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(11) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}d\theta$
(12) $S = \int_a^b 2\pi(x \text{ or } y)ds$
(13) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(14) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(15) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(16) $A = \int_a^b \frac{1}{2}r^2 d\theta$
(17) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(18) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$