MATH 166
SPRING 2012
EXAM 3

1. (48 pt) Determine if the following series converge of diverge.

$$\text{a)} \sum_{n=2}^{\infty} 2^{-\ln(n)} \qquad \text{b)} \sum_{n=1}^{\infty} \frac{\tan(\frac{1}{n}) - \sin(\frac{1}{n})}{n} \qquad \text{c)} \sum_{n=1}^{\infty} \frac{n^{n^2}}{(n!)^{2n}}$$

$$\text{d)} \sum_{n=2}^{\infty} (-1)^n \frac{f(n)}{n}, \text{ where } f(x) \text{ is decreasing and } \lim_{x\to\infty} f(x) = 1.$$

$$\text{e)} \sum_{n=3}^{\infty} \frac{5^{2n}(n!)^3}{(3n)!} \qquad \text{f)} \sum_{n=2}^{\infty} \frac{\sqrt[5]{n^6 + n^3 + 1}}{\sqrt[10]{3n^{21} + n^{20} - n^7 - 1}}$$

2. (16 pt) Consider the following sequences.

a) Show that the sequence defined by $a_1 = 1,\ a_{n+1} = 2 - \frac{1}{a_n + 1},\ n \geq 1$ converges.
b) Show that the $\{\frac{2n+3\sin(n!)}{3n+1}\}$ converges.

3. (16 pt) Consider the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$.

a) Show that this series converges.
b) How many terms are necessary to ensure that approximation $s \approx s_n$ is within $\frac{1}{100}$ of the sum of the series.

4. (12 pt) Find the center, radius, and interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{n^2(2x-4)^{2n}}{4^n(n^4+1)}.$$

5. (10 pt) Find

$$\int_1^{\infty} \frac{e^{-\frac{1}{x^2}}}{x^2} dx$$

with error no more than $\frac{1}{1000}$.

6. (8 pt) Suppose that $\sum_{n=1}^{\infty} a_n$ is a convergent positive term series and $f(x)$ is a nonnegative function that is differentiable at 0 with $f(0) = 0$. Determine the status of the series $\sum_{n=0}^{\infty} f(a_n)$.

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(6) $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(7) $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
(8) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(9) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(10) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(11) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2} dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2} d\theta$
(12) $S = \int_a^b 2\pi(x \text{ or } y) ds$
(13) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(14) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(15) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(16) $A = \int_a^b \frac{1}{2} r^2 d\theta$
(17) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(18) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$