1. (48 pt) Determine if the following series converge of diverge.

   a) $\sum_{n=2}^{\infty} 2^{-\ln(n)}$
   b) $\sum_{n=1}^{\infty} \frac{\tan\left(\frac{1}{n}\right) - \sin\left(\frac{1}{n}\right)}{n}$
   c) $\sum_{n=1}^{\infty} \frac{n^2}{(n!)^2}$
   d) $\sum_{n=2}^{\infty} (-1)^n \frac{f(n)}{n}$, where $f(x)$ is decreasing and $\lim_{x \to \infty} f(x) = 1$.
   e) $\sum_{n=3}^{\infty} \frac{5^{2n}(n!)^3}{(3n)!}$
   f) $\sum_{n=2}^{\infty} \frac{\sqrt[n]{n^6 + n^3 + 1}}{\sqrt[4]{3n^2 + n^2 + n^4 - n - 1}}$

2. (16 pt) Consider the following sequences.

   a) Show that the sequence defined by $a_1 = 1$, $a_{n+1} = 2 - \frac{1}{a_n+1}$, $n \geq 1$ converges.
   b) Show that the $\left\{\frac{2n+3\sin(n!)^3}{3n+1}\right\}$ converges.

3. (16 pt) Consider the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$.

   a) Show that this series converges.
   b) How many terms are necessary to ensure that approximation $s \approx s_n$ is within $\frac{1}{100}$ of the sum of the series.

4. (12 pt) Find the center, radius, and interval of convergence of the power series

   $$\sum_{n=0}^{\infty} \frac{n^2(2x - 4)^{2n}}{4^n(n^4 + 1)}$$

5. (10 pt) Find

   $$\int_{1}^{\infty} e^{-\frac{1}{x^2}} \frac{1}{x^2} \, dx$$

   with error no more than $\frac{1}{100}$.

6. (8 pt) Suppose that $\sum_{n=1}^{\infty} a_n$ is a convergent positive term series and $f(x)$ is a nonnegative function that is differentiable at 0 with $f(0) = 0$. Determine the status of the series $\sum_{n=0}^{\infty} f(a_n)$. 

(1) \( \sin(2x) = 2 \sin(x) \cos(x) \)
(2) \( \cos(2x) = \cos^2(x) - \sin^2(x) \)
(3) \( \cos^2(x) = \frac{1}{2} + \frac{1}{2} \cos(2x) \)
(4) \( \sin^2(x) = \frac{1}{2} - \frac{1}{2} \cos(2x) \)
(5) \( e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \)
(6) \( \sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \)
(7) \( \cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \)
(8) \( |EM| \leq \frac{K(b-a)^3}{24n^2} \)
(9) \( |ET| \leq \frac{K(b-a)^3}{12n^2} \)
(10) \( |ES| \leq \frac{K(b-a)^5}{180n^4} \)
(11) \( L = \int_a^b \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx = \int_a^b \sqrt{ \left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 } \, dt = \int_a^b \sqrt{r^2 + \left( \frac{dr}{d\theta} \right)^2} \, d\theta \)
(12) \( S = \int_a^b 2\pi x \, ds \)
(13) \( \int_{n+1}^{\infty} f(x) \, dx \leq R_n \leq \int_n^{\infty} f(x) \, dx \)
(14) \( \bar{x} = \frac{1}{A} \int_a^b x(f(x) - g(x)) \, dx \)
(15) \( \bar{y} = \frac{1}{2\pi} \int_a^b (f(x))^2 - (g(x))^2 \, dx \)
(16) \( A = \int_a^b \frac{1}{2} r^2 \, d\theta \)
(17) \( \int \sec(x) \, dx = \ln |\sec(x) + \tan(x)| + c \)
(18) \( \int \sec^3(x) \, dx = \frac{1}{2} \sec(x) \tan(x) + \frac{1}{2} \ln |\sec(x) + \tan(x)| + c \)