MATH 166
SPRING 2013
EXAM 3

1. (48 pt) Determine if the following series converge of diverge.

$$\text{a)} \sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n^3+2}}{\sqrt[5]{4n^{13}+n^9+5n^2+3}} \qquad \text{b)} \sum_{n=0}^{\infty}\left(\frac{(n!)^2}{(2n)!}\right)^n \qquad \text{c)} \sum_{n=1}^{\infty}\frac{n^k}{2^n}$$

$$\text{d)} \sum_{n=2}^{\infty} n^k \tan\left(\frac{1}{n^{k+2}}\right), k > -2 \qquad \text{e)} \sum_{n=1}^{\infty}(-1)^n \frac{\ln(n)}{n^2+1} \qquad \text{f)} \sum_{n=1}^{\infty}(-1)^n n \sin\left(\frac{1}{n}\right)$$

2. (16 pt) Determine if the following sequences converge or diverge.

a) $\{\frac{(n!)^2}{(2n)!}\}_{n=0}^{\infty}$ (hint: first consider $\sum_{n=0}^{\infty}\frac{(n!)^2}{(2n)!}$).
b) The sequence determined by $a_1 = 0$ and $a_{n+1} = 1 + \sqrt{1+a_n}$ for all $n \geq 1$.

3. (12 pt) Consider the series $\sum_{n=1}^{\infty}\frac{1}{n^2+1}$ and $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n^2+1}$.

a) How many terms are needed so that the estimate $\sum_{n=1}^{\infty}\frac{1}{n^2+1} \approx s_n$ has error less that $\epsilon > 0$.
b) How many terms are needed so that the estimate $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n^2+1} \approx s_n$ has error less that $\epsilon > 0$.

4. (10 pt) Find all values of x such that the series

$$\sum_{n=1}^{\infty}\frac{1}{n2^n x^n}$$

converges.

5. (16 pt) Consider the function $f(x) = \tan^{-1}(x)$.

a) Find a series representation for $f(x)$.
b) What is the interval of convergence of this series?
c) Use this series to find an infinite series for $\tan^{-1}(\frac{1}{2})$.
d) How many terms of this series are needed to estimate the infinite sum with error less than $\frac{1}{1000}$?

6. (8 pt) Let $\sum_{n=1}^{\infty} a_n$ be a positive term, convergent series and $b > 1$. Show that the series $\sum_{n=1}^{\infty}(b^{a_n} - 1)$ is convergent

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(6) $\sin(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(7) $\cos(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$
(8) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(9) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(10) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(11) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}d\theta$
(12) $S = \int_a^b 2\pi(x \text{ or } y)ds$
(13) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(14) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(15) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(16) $A = \int_a^b \frac{1}{2}r^2 d\theta$
(17) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(18) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$