MATH 166
SPRING 2009
EXAM 3A

1. (24 pt) Determine if the following sequences converge or diverge.

a) $\{\frac{\sqrt{n}\ln(n)}{(2n+1)}\}$ b) $\{f(n)\}$ where $f(x)$ is a decreasing function with $0 \leq f(x) \leq 1$

c) $\{(-1)^n \frac{n!n^2}{(n+2)!}\}$ d) $\{\sqrt[3]{6}, \sqrt[3]{6+\sqrt[3]{6}}, \sqrt[3]{6+\sqrt[3]{6+\sqrt[3]{6}}}, \cdots\}$

2. (36 pt) Determine if the following series converge or diverge.

a) $\sum_{n=1}^{\infty}(-1)^n \frac{n^2+1}{3n^2+90n}$ b) $\sum_{n=1}^{\infty} \frac{n\sin(n!)\tan^{-1}((3n)^n)}{n^4+n}$ c) $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^2+1}}{\sqrt[9]{(n^8+1)^2}}$

d) $\sum_{n=1}^{\infty}(\frac{1}{a^2+1})^{n^2}$, $a \neq 0$ e) $\sum_{n=0}^{\infty} \frac{5^n(n+1)!n!}{(2n)!}$ f) $\sum_{n=3}^{\infty}(\frac{1}{n} - \frac{1}{n+2})$

3. (12 pt) Let $\{a_n\}$ be a sequence of positive real numbers.

a) First suppose that $\lim_{n\to\infty} \sqrt[n]{a_n} = \frac{3}{4}$. Find $\lim_{n\to\infty} a_n$ and justify your answer.

b) Now consider the series $\sum_{n=1}^{\infty} a_n$. Let s_n denote the n^{th} partial sum of the series. If the sequence $\{s_n\}$ is bounded above (that is, there is a number B such that $s_n \leq B$ for all n), find $\lim_{n\to\infty} a_n s_n$.

4. (18 pt) Let k be a fixed positive integer and consider the series

$$\sum_{n=2}^{\infty}(-1)^n \frac{1}{(\ln(n))^{\frac{1}{k}}}.$$

a) Show that the series converges.

b) Find a formula for the number of terms needed to ensure that the approximation $s \approx s_n$ has error less than or equal to ϵ where $\epsilon > 0$ is a real number.

c) Explain why using the approximation $s \approx s_n$ may not be practical even for modestly small values of k (hint: roughly how big is n if $k = 1$ and $\epsilon = \frac{1}{100}$).

5. (12 pt) Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$.

a) Show that this series converges.

b) Find a formula for the number of terms needed to ensure that the approximation $s \approx s_n$ has error less than or equal to ϵ where $\epsilon > 0$ is a real number.

6. (8 pt) Suppose that two positive term series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ can be successfully limit compared (in the sense that $\lim_{n\to\infty} \frac{b_n}{a_n} = c$ where $0 < c < \infty$). Show that if the ratio test gives 1 for the series $\sum_{n=1}^{\infty} a_n$ then it also gives 1 for $\sum_{n=1}^{\infty} b_n$.