MATH 166
SPRING 2009
EXAM 3B

1. (24 pt) Find the center, radius, and interval of convergence of the following series.

$$\text{a)} \sum_{n=0}^{\infty} \frac{(2x-1)^{2n}}{na^n}, a>0 \qquad \text{b)} \sum_{n=1}^{\infty} \frac{x^{n^2}}{n^n}$$

$$\text{c)} \sum_{n=2} (-1)^n \frac{\tan^{-1}(n)(x+3)^n}{n\ln(n)} \qquad \text{d)} \sum_{n=0}^{\infty} 2^n x^{2^n}$$

2. (36 pt) Consider the series

$$\sum_{n=0}^{\infty} \frac{3^n \ln(n)}{n} (\frac{1}{x})^n.$$

   a) Find the values of x for which this series converges.
   b) Find the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{3^n \ln(n)}{n} x^{2n}$

3. (12 pt) Write the following series as a more familiar function.
   a) $\sum_{n=0}^{\infty}(n+1)x^{2n}$ (hint: first consider $\sum_{n=0}^{\infty}(n+1)x^n$).
   b) $\sum_{n=0}^{\infty} \frac{x^{4n}}{n!}$.
   c) $\sum_{n=1}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!(2n+1)}$.

4. (18 pt) Consider the infinite series

$$\sum_{n=0}^{\infty} \sin^n(\theta)$$

   a) Find all values of θ for which this series converges.
   b) Find the sum of this series (for values of θ for which it converges).

5. (12 pt) Consider the function

$$f(x) = \frac{1}{a^k + x^k}, a > 1.$$

   a) Find a Maclaurin series for $f(x)$ and determine its interval of convergence.
   b) Show that $\int_0^{\frac{a}{2}} \frac{dx}{a^k+x^k} \approx a$ with error no more than $\frac{1}{4}$.

6. (8 pt) Approximate the integral

$$\int_0^{\frac{1}{2}} x^2 \cos(x^2) dx$$

with error less than or equal to $\frac{1}{1000}$.