1. (48 pt) Determine if the following series converge or diverge.

   a) \( \sum_{n=1}^{\infty} \frac{1}{(\ln(n))^\ln(n)} \)
   b) \( \sum_{n=0}^{\infty} (-1)^n \frac{\ln(n)}{3n+1} \)
   c) \( \sum_{n=0}^{\infty} \frac{(3n)!}{n!(2n)!2^{3n}} \)
   d) \( \sum_{n=1}^{\infty} \sin\left(\frac{1}{n^3}\right) \)
   e) \( \sum_{n=1}^{\infty} \frac{\sqrt[10]{16n^{10} + 2n^5 + 1}}{\sqrt[18]{32n^{18} + n^3 + n + 7}} \)
   f) \( \sum_{n=2}^{\infty} (\sqrt[9]{9} - \sqrt[9]{9}) \)

2. (20 pt) Consider the following sequence

   \[ a_{n+1} = \sqrt{2a_n} - 1, \quad n \geq 1, \quad a_1 > \frac{1}{2}. \]

   a) If \( a_1 \neq 1 \) show that this sequence is always decreasing.
   b) Show if \( a_1 > 1 \) show that this sequence has a floor of 1.
   c) Explain why this sequence converges if \( a_1 > 1 \).
   d) If this sequence converges, what is its limit?
   e) What happens if \( \frac{1}{2} < a_1 < 1 \)?

3. (12 pt) Determine if the following sequences converge or diverge.

   a) \( (a, \sin(a), \sin(\sin(a)), \ldots) \)
   b) \( \left(\frac{(-1)^n(n^2 + n\sin(n))}{n^2 + 1}\right)_{n=1}^{\infty} \)

4. (20 pt) Consider the series

   \( \sum_{n=1}^{\infty} \frac{4n}{(n^2 + 1)^3} \) and \( \sum_{n=1}^{\infty} (-1)^n \frac{4n}{(n^2 + 1)^3} \)

   a) Show that the first series converges.
   b) How many terms are required so that the approximation \( s \approx s_n \) has error less than or equal to \( \frac{1}{100} \).
   c) Show that the second series converges.
   d) How many terms are required so that the approximation \( s \approx s_n \) has error less than or equal to \( \frac{1}{100} \).

5. (10 pt) Consider the series

   \( \sum_{n=0}^{\infty} a_n \),

   and suppose that the partial sums satisfy the formula

   \( s_n = 3n\sin\left(\frac{2}{n}\right) \).

   a) Does this series converge? If so, what is its sum?
   b) What is \( \lim_{n \to \infty} a_n \) or is there not enough information to tell?