MATH 166
SUMMER 2011
EXAM 5

1. (30 pt) Find the center, radius and interval of convergence of the following power series.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n(x-1)^n}{(n+1)5^{n+1}}$ b) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{kn}}{n^n}$, k a positive integer. c) $\displaystyle\sum_{n=1}^{\infty} \frac{9^n}{n^2}(2x+4)^{(2n+4)}$

2. (10 pt) Consider the power series

$$\sum_{n=1}^{\infty}(x+1)^{n^2}$$

Find the center, radius, and interval of convergence of this series (hint: compare this to the easier to handle series $\sum_{n=1}^{\infty}(x+1)^n$).

3. (20 pt) Find Maclaurin series for the following functions.
   a) $f(x) = x^2e^{-3x^4}$.
   b) $f(x) = \ln(1-x^3)$.

4. (15 pt) Estimate

$$\int_0^{\frac{1}{10}} \frac{x}{1+x^6}\,dx$$

with error less than or equal to $\frac{1}{10^9}$.

5. (20 pt) For this problem, we recall that

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, -1 < x < 1.$$

   a) Find a Maclaurin series for $f(x) = \tan^{-1}(x)$.
   b) For this part, assume that $x \geq 0$ and find a series for $\tan^{-1}(\sqrt{x})$.
   c) Again, assuming $x > 0$ find a series for $\sqrt{x}\tan^{-1}(\sqrt{x})$ (note this one is, in fact, a power series).
   d) Does the series from part c) equal $\sqrt{x}\tan^{-1}(\sqrt{x})$? Why or why not?

6. (15 pt) For this problem, recall that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.
   a) Show that $e = \sum_{n=0}^{\infty} \frac{1}{n!}$.
   b) Show that the error induced in approximating $e \approx 1 + 1 + \frac{1}{2!} + \cdots + \frac{1}{n!}$ is less than or equal to $\frac{n+2}{(n+1)!(n+1)}, n \geq 0$. (Hint: The actual error is $\frac{1}{(n+1)!} + \frac{1}{(n+2)!} + \cdots$; bound this by a convenient geometric series.)
   c) How many terms are needed so that the approximation above has error no more than $\frac{1}{100}$?

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(6) $\sin(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(7) $\cos(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$
(8) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(9) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(10) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(11) Force=(pressure)(area) and pressure=ρ(depth).
(12) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}d\theta$
(13) $S = \int_a^b 2\pi(x \text{ or } y)ds$
(14) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(15) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(16) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(17) $A = \int_a^b \frac{1}{2}r^2 d\theta$
(18) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(19) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$
(20) The surface area of a cone: $A = \pi r L$ where r is the radius and L is the slant height.