MATH 166
SUMMER 2011
EXAM 6

1. (48 pt) Consider the parametric equations $x = t^4 - 8t^2$ and $y = t^3 - 12t$.
   a) Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$ and the intervals of increase of x and y (a table would be appropriate for this).
   b) Sketch the graph of this parametric equation.
   c) Find the area enclosed by the "loop".
   d) Find $\frac{dy}{dx}$ and determine the values of t for which the slope of the tangent is positive and where it is negative. Does this match your graph?
   e) Find $\frac{d^2y}{dx^2}$ and determine the values of t for which concavity is positive and where it is negative. Does this match your graph?
   f) Find the length of this curve $-2 \leq t \leq 2$.

2. (24 pt) Consider the polar equation $r = 2 + \sin(\frac{1}{2}\theta)$.
   a) Sketch the graph of this polar equation.
   b) Find the area of the inside loop.
   c) Find the area enclosed by the area enclosed by this curve, excluding the inner loop.

3. (24 pt) Consider the polar equation $r = \theta$.
   a) Sketch this curve.
   b) Find the length of this curve $0 \leq \theta \leq \frac{\pi}{2}$.
   c) Find the area enclosed by this curve $0 \leq \theta \leq \frac{\pi}{2}$.

4. (14 pt) Consider the polar equation $r = k\csc(\theta)$, where $k > 0$ is constant.
   a) Find the area enclosed by this curve from $\theta = \tan^{-1}(\frac{k}{b})$ to $\theta = \tan^{-1}(\frac{k}{a})$ where $b > a > 0$.
   b) Find the area under this curve for $a \leq x \leq b$ ($b > a > 0$).

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(6) $\sin(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(7) $\cos(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$
(8) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(9) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(10) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(11) Force=(pressure)(area) and pressure=ρ(depth).
(12) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}d\theta$
(13) $S = \int_a^b 2\pi(x \text{ or } y)ds$
(14) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(15) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(16) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(17) $A = \int_a^b \frac{1}{2}r^2 d\theta$
(18) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(19) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$
(20) The surface area of a cone: $A = \pi r L$ where r is the radius and L is the slant height.
(21) $\frac{dy}{dx} = \frac{\frac{d}{dt}(y)}{\frac{dx}{dt}} = \frac{\frac{dr}{d\theta}\sin(\theta) + r\cos(\theta)}{\frac{dr}{d\theta}\cos(\theta) - r\sin(\theta)}$
(22) $\int \sec^5(x)\ dx = \frac{1}{4}\sec^3(x)\tan(x) + \frac{3}{8}\sec(x)\tan(x) + \frac{3}{8}\ln(|\sec(x) + \tan(x)|) + c$