MATH 166
SUMMER 2011
QUIZ 11

1. Suppose that we want to estimate $\int_a^b f(x)dx$ and suppose that the maximum value of $|f''(x)|$ is equal to the maximum value of $|f^{(4)}(x)|$ on $[a, b]$.

a) (5 pt) Find a formula for the smallest value of n for which the error bound for Simpson's rule is smaller than the error bound for the trapezoid rule.

b) (5 pt) If $a = 0$, $b = \sqrt{135}$, how many boxes do you need so that the error bound for the trapezoid rule is no more than the value from part a)?

c) (5 pt) Answer part b) again for Simpson's rule.

2. (5 pt) Suppose that you are estimating $\int_0^2 f(x)dx$ using Simpson's rule. You find that the fourth derivative of $f(x)$ is given by e^{x^3-3x}. Find the appropriate value of K for computing the error bound.

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $\sin(A)\cos(B) = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
(6) $\sin(A)\sin(B) = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
(7) $\cos(A)\cos(B) = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$
(8) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(9) $\sin(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(10) $\cos(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$
(11) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(12) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(13) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(14) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2} dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2} d\theta$
(15) $S = \int_a^b 2\pi(x \text{ or } y) ds$
(16) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(17) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(18) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(19) $A = \int_a^b \frac{1}{2} r^2 d\theta$
(20) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(21) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$
(22) If $t = \tan(\frac{x}{2})$ then $\sin(x) = \frac{2t}{t^2+1}, \cos(x) = \frac{1-t^2}{t^2+1}, dx = \frac{2dt}{t^2+1}$