MATH 166
SUMMER 2011
QUIZ 13

1. (5 pt) Suppose $f(x)$ is a differentiable function and $|f'(x)| \leq m$ for all $a \leq x \leq b$. Show that the length of the curve $y = f(x), a \leq x \leq b$ satisfies

$$L \leq \sqrt{m^2 + 1}(b - a).$$

2. (5 pt) Find the surface area obtained when the curve $y = e^x, 0 \leq x \leq a$ is revolved about the $x-$axis.

3. In this problem, we wish to find the surface area of a sphere.
   a) (5 pt) Set up an integral that will give us the area that we seek.
   b) (5 pt) Evaluate the integral.

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$

(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$

(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$

(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$

(5) $\sin(A)\cos(B) = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$

(6) $\sin(A)\sin(B) = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

(7) $\cos(A)\cos(B) = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

(8) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

(9) $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

(10) $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$

(11) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$

(12) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$

(13) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$

(14) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2} dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2} d\theta$

(15) $S = \int_a^b 2\pi(x \text{ or } y) ds$

(16) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$

(17) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$

(18) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2] dx$

(19) $A = \int_a^b \frac{1}{2} r^2 d\theta$

(20) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$

(21) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$

(22) If $t = \tan(\frac{x}{2})$ then $\sin(x) = \frac{2t}{t^2+1}, \cos(x) = \frac{1-t^2}{t^2+1}, dx = \frac{2dt}{t^2+1}$