MATH 166
SUMMER 2011
QUIZ 17

1. Determine if the following sequences converge.

   a) (5 pt) $(\frac{\tan^{-1}(n)}{2n+1})_{n=1}^{\infty}$.

   b) (5 pt) $(\ln(2n\tan(\frac{3}{n})))_{n=1}^{\infty}$.

   c) (5 pt) $(\sqrt{n^2+1}-\sqrt{n^2+4n+1})_{n=1}^{\infty}$.

2. (5 pt) Let $f(x)$ be a function with the property that $f'(x) > 0$ for all $x > 0$. If $f(x)$ has a horizontal asymptote (to the right) then show that the sequence $(a_n)_{n=1}^{\infty} = (f(n))_{n=1}^{\infty}$ converges.

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $\sin(A)\cos(B) = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
(6) $\sin(A)\sin(B) = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
(7) $\cos(A)\cos(B) = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$
(8) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(9) $\sin(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(10) $\cos(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$
(11) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(12) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(13) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(14) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}d\theta$
(15) $S = \int_a^b 2\pi(x \text{ or } y)ds$
(16) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(17) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(18) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(19) $A = \int_a^b \frac{1}{2}r^2 d\theta$
(20) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(21) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$
(22) If $t = \tan(\frac{x}{2})$ then $\sin(x) = \frac{2t}{t^2+1}, \cos(x) = \frac{1-t^2}{t^2+1}, dx = \frac{2dt}{t^2+1}$