MATH 166
SPRING 2003
FINAL EXAM

1. (32 pt) Evaluate the following integrals.

a) $\int_0^2 \sqrt{x^2+2x}dx$ b) $\int_0^\infty \frac{e^x}{e^{2x}+1}dx$ c) $\int x^2e^{2x}dx$

d) $\int_0^{\frac{\pi}{4}} \tan^{-1}(x)dx$

2. (18 pt) Determine if the following series converge or diverge.

a) $\sum_{n=1}^{\infty}(-1)^n\frac{n}{2n+1}$ b) $\sum_{n=1}^{\infty}\frac{\ln(n)}{n^2 2^n}$ c) $\sum_{n=2}^{\infty}\frac{\sin(e^n)}{n^2}$

3. (10 pt) Sketch the curve defined by the parametric equations $x = t^3 - 3t^2$, $y = t^4 - 4t^3$.

4. (6 pt) Find the center, radius, and interval of convergence for the power series

$$\sum_{n=1}^{\infty}\frac{x^{2n}}{n!}.$$

Write this series as a more familiar function.

5. (5 pt) Consider the polar equation $r = a\cos(\theta)$, $a > 0$. Find the length of this curve and the area enclosed by this curve.

6. (5 pt) Find the area enclosed by the inner loop in the graph of the polar equation $r = \frac{1}{2} + \cos(\theta)$.

7. (8 pt) Consider the ellipse defined by the parametric equations $x = a\cos(t), y = b\sin(t)$, $a, b > 0$, $0 \le t \le 2\pi$.

a) Find the area enclosed by this ellipse.
b) Find the volume obtained when the upper half of this ellipse is revolved about the x-axis.

8. (6 pt) A region that lies in the plane is contained in the first quadrant and its centroid is located at the point $(2, 2)$. When this region is revolved about the line $y = -x$, the solid obtained has a volume of $24\sqrt{2}\pi$. What is the area of this region?

9. (6 pt) A spring made of unusual material obeys a modified Hooke's Law. The force required to hold this spring at a length x beyond its natural length is given by $F(x) = 2x\sin(x)$ for $0 \le x \le \frac{\pi}{4}$ (in feet). Find the work done in stretching this spring from its natural length of 1 foot to a length of 1.5 feet.

10. (8 pt) Find the surface area obtained when the upper half of the circle $x^2 + (y-a)^2 = R^2$, $0 < R \le a$, is revolved about the x-axis.

11. (6 pt) Consider the function given by $f(x) = \sum_{n=1}^{\infty}(-1)^n\frac{x^n}{n2^n}$. Estimate $\int_0^{\frac{1}{2}} f(x)dx$ with error less than or equal to $\frac{1}{1000}$. You may assume that the radius of convergence of the series is 2.