1. (32 pt) Evaluate the following integrals.

$$\text{a) } \int \frac{x^2\,dx}{2x+1} \qquad \text{b) } \int \frac{e^x\,dx}{\sqrt{e^{2x}+1}} \qquad \text{c) } \int_0^\infty \frac{e^{-\frac{1}{x}}}{x^2}\,dx \qquad \text{d) } \int_1^e \ln(x)\,dx$$

2. (10 pt) Determine if the following sequences converge or diverge.

$$\text{a) } \left\{ \frac{\sin((n!)^n)}{\sqrt{2n}} \right\}_{n=1}^\infty \qquad \text{b) } \Big\{ \sin(2), \sin(\sin(2)), \sin(\sin(\sin(2))), \cdots \Big\}$$

3. (15 pt) Determine if the following series converge or diverge.

$$\text{a) } \sum_{n=1}^\infty \frac{n^2+2}{\sqrt[3]{n^{10}+8n^3+2}} \qquad \text{b) } \sum_{n=1}^\infty \left(\frac{3n+1}{4n-5}\right)^{2n+3} \qquad \text{c) } \sum_{n=0}^\infty (-1)^n \frac{n^2}{2^n}$$

4. (10 pt) Consider the polar curve $r = \sin(\frac{1}{2}\theta)$.
   a) Sketch this curve.
   b) Find the area enclosed by the two inner loops.

5. (15 pt) Consider the curves $y = x$ and $y = x\sin(x)$, $0 \le x \le \frac{\pi}{2}$.
   a) Find the area of the region bounded by these two curves ($0 \le x \le \frac{\pi}{2}$).
   b) Find the volume obtained when this region is revolved about the $y-$axis.
   c) Set up (but do not evaluate) an integral(s) that expresses the surface area of the solid obtained when this region is revolved about the $x-$axis.

6. (5 pt) Suppose that you wish to approximate $\int_1^3 f(x)\,dx$ using Simpson's rule. You find that $f^{(4)}(x) = \frac{x}{x^2+4}$. Find the smallest n so that the approximation S_n has error less than or equal to $\frac{1}{1800}$.

7. (8 pt) Sketch the graph of the parametric curve given by $x = \frac{t}{t^2+1}$, $y = t^3 - 12t$.

8. (5 pt) Consider the portion of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ that is contained in the first quadrant. When you revolve this region about the $x-$axis you get a semiellipsoid of volume $V_x = \frac{4}{3}\pi ab^2$ and when you revolve this region about the $y-$axis you get a semiellipsoid of volume $V_y = \frac{4}{3}\pi a^2 b$. Use this to show that the centroid of the region lies on the line $y = \frac{b}{a}x$.

9. (5 pt) Suppose that you pull a bucket containing 75 pounds of water up a 100 foot well. If the rope you are using weighs 2 pounds per foot, how much work is done in hoisting the bucket all the way up the well?

10. (5 pt) Write the function $f(x) = \frac{1}{x}$ as a Taylor series centered at 1 and determine its interval of convergence.