

MATH 166
SPRING 2005
FINAL EXAM

1. (32 pt) Evaluate the following integrals.

a) $\int x^3 \sqrt{x^2 + 1} dx$ b) $\int \frac{x^2}{\sqrt{x^2 - 4}} dx$ c) $\int_0^\infty \frac{2x}{x^4 + 2x^2 + 2} dx$ d) $\int_0^1 x \tan^{-1}(x) dx$

2. (8 pt) Determine if the following sequences converge or diverge.

a) $\left\{ 2n \sin\left(\frac{5}{n}\right) \right\}_{n=1}^\infty$ b) $\left\{ 2^{\left(\sum_{k=1}^n \frac{1}{k^3}\right)} \right\}_{n=1}^\infty$

3. (12 pt) Determine if the following series converge or diverge.

a) $\sum_{n=1}^\infty \frac{2^n (n!) ((n+1)!)}{(2n)!}$ b) $\sum_{n=1}^\infty \frac{\sqrt[3]{n} + 1}{n\sqrt{n+1}}$ c) $\sum_{n=1}^\infty (-1)^n \frac{\sin(n+4)}{n^2}$

4. (8 pt) Sketch the polar curve $r = \sin^2(\theta)$ and find the area that it encloses.

5. (8 pt) Consider the portion of the parabola $2x = y^2$ that lies in the first quadrant ($0 \leq y \leq 2$).

- a) Set up the integral to find the length of this curve (you need not evaluate the integral).
- b) Find the surface area obtained when this curve is revolved about the x -axis.

6. (15 pt) Consider the curves $f(x) = 3x$ and $g(x) = x^3 - x$, $0 \leq x \leq a$ where a is the first positive x -coordinate of the intersection of the two curves. We will call the region bounded by these two curves \mathfrak{R} .

- a) Find the area of \mathfrak{R} .
- b) Find the volume obtained when \mathfrak{R} is revolved about the y -axis.
- c) Find the x -coordinate of the centroid of \mathfrak{R} .

7. (10 pt) Consider the curve given by the parametric equations $x = t^2$ and $y = t^3 - 3t$.

- a) Sketch this curve.
- b) Find the total area enclosed by the loop in this curve (hint: where does this curve intersect the x -axis?).

8. (6 pt) A glass aquarium has a rectangular side of height 4 feet and width 10 feet and is filled to the top with water. If water weighs 62.5 pounds per cubic foot, find the force due to hydrostatic pressure on this side (recall that pressure times area is force and pressure is proportional to depth).

9. (6 pt) Find the center, radius, and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{\ln(n)(x-4)^n}{n3^{2n}}.$$

10. (5 pt) Use a series to estimate $\tan^{-1}\left(\frac{1}{2}\right)$ with error less than $\frac{1}{500}$.