MATH 166
SPRING 2006
FINAL EXAM

1. (24 pt) Evaluate the following integrals.

$$a) \int \sqrt{a^2x^2+1}\, dx,\ a \neq 0 \qquad b) \int x^3 e^{x^2}\, dx \qquad c) \int_1^{\infty} \frac{dx}{x^4+x^2}$$

2. (8 pt) Determine if the following sequences converge or diverge.

$$a) \left\{ (-1)^n \frac{n}{2n+1} \right\}_{n=1}^{\infty} \qquad b) \left\{ r, r^2, r^3, \cdots \right\}_{n=1}^{\infty} \text{ where } r \text{ is a root of the polynomial } x^3+x-1.$$

3. (12 pt) Determine if the following series converge or diverge.

$$a) \sum_{n=1}^{\infty} \frac{2n\cos(n)}{n^3+7} \qquad b) \sum_{n=2}^{\infty} \frac{2^n+10}{n3^n} \qquad c) \sum_{n=1}^{\infty} (-1)^n \frac{e^{-n}}{\sin(e^{-n})}$$

4. (25 pt) Consider the functions $f(x) = x^2$ and $g(x) = x^3$, $0 \le x \le 1$.
   a) Locate the centroid of the region bounded by $f(x)$ and $g(x)$.
   b) Find the volume obtained when this region is revolved about the $x-$axis.
   c) Find the volume obtained when this region is revolved about the $y-$axis.
   d) Find the length of $f(x)$, $0 \le x \le 1$.
   e) Find the surface area obtained when $g(x)$ (for $0 \le x \le 1$) is revolved about the $y-$axis.

5. (9 pt) Consider the polar curves $r = 2\cos(\frac{1}{3}\theta)$ and $r = \theta$.
   a) Sketch the curve $r = 2\cos(\frac{1}{3}\theta)$.
   b) Find the area of the inner loop of $r = 2\cos(\frac{1}{3}\theta)$.
   c) Find the length of the curve $r = \theta$, $0 \le \theta \le \pi$.

6. (10 pt) Consider the curve given by the parametric equations $x = \frac{t^2}{t^2+1}$ and $y = \frac{t^3}{t^2+1}$. For your convenience, $\frac{dx}{dt} = \frac{2t}{(t^2+1)^2}$ and $\frac{dy}{dt} = \frac{t^4+3t^2}{(t^2+1)^2}$.
   a) Sketch this curve and pay special attention to what happens at the origin (what can you say about the limit of the slope of the tangent to this curve as t approaches 0 from both sides?).
   b) Write an integral that expresses the area bounded by this curve and the line $x = 1$. Is this area finite?

7. (10 pt) A sphere of radius 1 has a volume $\frac{4}{3}\pi$. You wish to make a "napkin ring" out of this by drilling a hole of radius r all the way through the sphere. How big should r be so that the volume of the resulting "napkin ring" is exactly π?

8. (6 pt) Find the center, radius, and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(2x-5)^{3n}}{8^{n+3}}.$$

9. (6 pt) Suppose that you wish to approximate $\int_0^2 f(x)dx$ using Simpson's Rule. You find that the fourth derivative of $f(x)$ is given by $f^{(4)}(x) = \frac{x}{x^2+1}$. Is using S_{10} good enough to ensure an error of less than $\frac{1}{100{,}000}$?