1. (15 pt) Determine if the following sequences converge or diverge.

   a) \( \lim_{n \to \infty} \frac{\sin(n!)}{\sqrt{n + 1}} \)
   
   b) \( \lim_{n \to \infty} \frac{1}{\tan^{-1}\left(\sum_{k=1}^{n} \frac{1}{2k}\right)} \)
   
   c) \( \{a_n\}_{n=1}^{\infty} \), where \( a_1 = 1 \) and \( a_{n+1} = \frac{1}{a_n + 1}, n \geq 1 \).

2. (20 pt) Determine if the following series converge or diverge.

   a) \( \sum_{n=2}^{\infty} \frac{1}{\ln(n)} \)
   
   b) \( \sum_{n=2}^{\infty} \frac{1}{(\ln(n))^n} \)
   
   c) \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{n} \tan^{-1}(n)} \)
   
   d) \( \sum_{n=1}^{\infty} \frac{\sin(n)}{n^2 + n} \)

3. (20 pt) Evaluate the following integrals.

   a) \( \int \sqrt{2x - x^2} \, dx \)
   
   b) \( \int_{0}^{\infty} \frac{e^x}{e^{2x} + 1} \, dx \)
   
   c) \( \int_{0}^{\sqrt{3}} \frac{x^3}{\sqrt{x^2 + 1}} \, dx \)
   
   d) \( \int \frac{\ln(x)}{x^2} \, dx \)

4. (5 pt) Sketch the curve defined by the parametric equations \( x = t^3 - 3t \) and \( y = t^3 - 12t \).

5. (5 pt) Consider the polar equation \( r = \frac{1}{2} + \sin(\theta) \).

   a) Sketch this curve.
   
   b) Find the area enclosed by the inner loop.

6. (10 pt) Consider an inverted cone with base (roof) radius \( R \) and height \( h \). Suppose that this container is filled with a liquid of density \( \rho \).

   a) Find a function \( p(x) \) that tells how much work is done in pumping \( x \) vertical feet of liquid out of the tank.
   
   b) Compute the average value of \( p(x) \) on the interval \([0, h]\).

7. (8 pt) Find the center, radius, and interval of convergence for the power series

   \[ \sum_{n=1}^{\infty} \frac{(-1)^n (2x - 4)^{2n}}{n3^n \ln(n)} \]

8. (7 pt) Find a Maclaurin series for the function

   \[ f(x) = \begin{cases} \frac{e^x - 1}{x}, & \text{if } x \neq 0; \\ 1, & \text{if } x = 0, \end{cases} \]

   and use this series to approximate \( \int_{-\frac{1}{2}}^{0} f(x) \, dx \) with error less than \( \frac{1}{500} \).

9. (5 pt) Find the length of the curve \( y = \ln(\cos(x)), 0 \leq x \leq \frac{\pi}{4} \).

10. (15 pt) Consider a sphere of radius \( R \) obtained by revolving the upper half-circle of radius \( R \) about the \( x \)-axis.

   a) Find the volume of the sphere.
   
   b) Find the surface area of the sphere.
   
   c) Locate the centroid of the upper half-circle.