MATH 166
SPRING 2009
FINAL EXAM

1. (15 pt) Determine if the following sequences converge or diverge.

$$a)\{\frac{\sin(n!)}{\sqrt{n+1}}\}_{n=1}^{\infty} \qquad b)\{\tan^{-1}(\sum_{k=1}^{n}\frac{1}{2^k})\}_{n=1}^{\infty} \qquad c)\{a_n\}_{n=1}^{\infty}, \text{where } a_1 = 1 \text{ and } a_{n+1} = \frac{1}{a_n+1}, n \geq 1.$$

2. (20 pt) Determine if the following series converge or diverge.

$$a)\sum_{n=2}^{\infty}\frac{1}{\ln(n^n)} \qquad b)\sum_{n=2}^{\infty}\frac{1}{(\ln(n))^n} \qquad c)\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}\tan^{-1}(n)} \qquad d)\sum_{n=1}^{\infty}\frac{\sin(n)}{n^2+n}$$

3. (20 pt) Evaluate the following integrals.

$$a)\int\sqrt{2x-x^2}dx \qquad b)\int_0^{\infty}\frac{e^x}{e^{2x}+1}dx \qquad c)\int_0^{\sqrt{3}}\frac{x^3}{\sqrt{x^2+1}}dx \qquad d)\int\frac{\ln(x)dx}{x^2}$$

4. (5 pt) Sketch the curve defined by the parametric equations $x = t^3 - 3t$ and $y = t^3 - 12t$.

5. (5 pt) Consider the polar equation $r = \frac{1}{2} + \sin(\theta)$.
   a) Sketch this curve.
   b) Find the area enclosed by the inner loop.

6. (10 pt) Consider an inverted cone with base (roof) radius R and height h. Suppose that this container is filled with a liquid of density ρ.
   a) Find a function $p(x)$ that tells how much work is done in pumping x vertical feet of liquid out of the tank.
   b) Compute the average value of $p(x)$ on the interval $[0, h]$.

7. (8 pt) Find the center, radius, and interval of convergence for the power series

$$\sum_{n=1}^{\infty}(-1)^n\frac{(2x-4)^{2n}}{n3^n\ln(n)}.$$

8. (7 pt) Find a Maclaurin series for the function

$$f(x) = \begin{cases} \frac{e^x-1}{x}, & \text{if } x \neq 0; \\ 1, & \text{if } x = 0, \end{cases}$$

and use this series to approximate $\int_{-\frac{1}{2}}^{0} f(x)dx$ with error less than $\frac{1}{500}$.

9. (5 pt) Find the length of the curve $y = \ln(\cos(x)), 0 \leq x \leq \frac{\pi}{4}$.

10. (15 pt) Consider a sphere of radius R obtained by revolving the upper half-circle of radius R about the $x-$axis.
    a) Find the volume of the sphere.
    b) Find the surface area of the sphere.
    c) Locate the centroid of the upper half-circle.