MATH 166
SPRING 2010
FINAL EXAM

1. (10 pt) Determine if the following sequences converge or diverge.

a)$\{n\tan(\frac{1}{n})\}_{n=1}^{\infty}$ b)$\{f(n), f(f(n)), f(f(f(n))), \cdots\}_{n=1}^{\infty}$ where $f(x)$ is a positive function such that $f(x) \leq x$ for all $x > 0$.

2. (24 pt) Determine if the following series converge or diverge.

a) $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{\ln(4n)}{n+2}$ b) $\displaystyle\sum_{n=1}^{\infty}\frac{n+1}{\sqrt[3]{5n^8+7n+2}}$ c) $\displaystyle\sum_{n=1}^{\infty}(\frac{1+3n^2}{2n^2+n+5})^{3n}$ d) $\displaystyle\sum_{n=1}^{\infty}\frac{\tan^{-1}(n!)}{\sqrt{n^3+1}}$

3. (24 pt) Evaluate the following integrals.

a) $\displaystyle\int\sqrt{x^2+2Rx+2R^2}dx$ b) $\displaystyle\int x^2\ln(x^2)dx$ c) $\displaystyle\int_0^{\infty}\frac{x^2+3}{x^4-1}dx$ d) $\displaystyle\int_0^{\pi^2}\sin(\sqrt{x})dx$

4. (12 pt) Consider the curve described by the parametric equations $x = \sqrt{t^2+1}$ and $y = t^3 - 3t$.

a) Sketch this curve.
b) Set up (do not evaluate) an integral that finds the area enclosed by the closed loop of this curve.
c) Set up (do not evaluate) an integral that finds the length of this loop.

5. (6 pt) Consider the polar equation $r = 2 + \cos(2\theta)$.

a) Sketch this curve.
b) Find the area enclosed by this curve.

6. (10 pt) A bucket full of liquid is being hauled up a well of depth D feet. The bucket weighs b pounds and the cable used to raise the bucket weighs c pounds per foot. If the bucket has a load of L pounds inside of it, compute the amount of work it takes to raise the bucket out of the well.

7. (12 pt) Consider the region bounded by the $x-$axis and the function $f(x) = \sin(x)$.

a) Find the area of this region.
b) Locate the centroid of this region.
c) Find the volume obtained when this region is revolved about the $y-$axis.
d) Find the volume obtained when this region is revolved about the line $y = 2$.

8. (6 pt) Find the Maclaurin series for $f(x) = \sin(x^3)$ and use this to estimate

$$\int_0^1 \sin(x^3)dx$$

with error less than $\frac{1}{1500}$.

9. (6 pt) Find all solutions to the differential equation

$$x\frac{dy}{dx} = y(x^2+1).$$

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(6) $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(7) $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
(8) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(9) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(10) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(11) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2} dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2} d\theta$
(12) $S = \int_a^b 2\pi(x \text{ or } y) ds$
(13) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(14) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(15) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(16) $A = \int_a^b \frac{1}{2} r^2 d\theta$
(17) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(18) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$