**MATH 166**
**SUMMER 2011**
**FINAL EXAM**

1. (32 pt) Evaluate the following integrals.

$$\text{a)} \int \frac{dx}{\sqrt{x^2-1}} \qquad \text{b)} \int e^{2x}\cos(3x)\ dx \qquad \text{c)} \int_0^1 x\tan^{-1}(x)\ dx$$
$$\text{d)} \int_0^\infty \frac{1}{\sqrt{x}(x+4)}\ dx$$

2. (24 pt) Determine if the following series converge or diverge.

$$\text{a)} \sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{n} \qquad \text{b)} \sum_{n=2}^{\infty} \frac{(2n)!}{n!n^n} \qquad \text{c)} \sum_{n=1}^{\infty} \frac{\sqrt{n^2+1}}{\sqrt[3]{n^8+6n^3+1}}$$

3. (15 pt) Consider the region bounded by the parabola $y = 2x - x^2$ and the line $y = x$.
   a) Find the area of this region.
   b) Find the $x-$coordinate of the centroid of this region.
   c) Find the $y-$coordinate of the centroid of this region.
   d) Find the volume obtained when this region is revolved about the $x-$axis.
   e) Find the volume obtained when this region is revolved about the $y-$axis.

4. (8 pt) Consider the parametric equations $x = \ln|t^2-1|$ and $y = \ln|t^2-4|$.
   a) Compute $\frac{dx}{dt}$ and $\frac{dy}{dt}$ and find where $x$ and $y$ are increasing and decreasing.
   b) Sketch this curve.

5. (8 pt) Consider the polar curve $r = e^\theta, 0 \le \theta \le \pi$.
   a) Find the length of this curve.
   b) Calculate the area enclosed by this curve.

6. (6 pt) For this problem, we consider an aquarium with depth $h$ and rectangular base of sides lengths $a, b > 0$. The aquarium is filled to the top with a liquid of density $\rho$.
   a) Find the total force due to hydrostatic pressue on the tank (this includes all four sides and the bottom).
   b) Find the work done in pumping all of the liquid out of the tank.

7. (9 pt) Consider the function $f(x) = \ln(1-x^3)$.
   a) Find the Maclaurin series for this function.
   b) Use this series to find $\ln(\frac{9}{8})$.
   c) Hw many terms are need so that the error of the approximation $s \approx s_n$ has error less or equal to $\frac{1}{1000}$.

8. (4 pt) Find the center, radius, and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(2x-6)^n}{3^n}.$$

9. (4 pt) Evaluate $\int_0^1 \sin(x^2)\ dx$ with error no more than $\frac{1}{100}$.

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $\sin(A)\cos(B) = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
(6) $\sin(A)\sin(B) = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
(7) $\cos(A)\cos(B) = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$
(8) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(9) $\sin(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(10) $\cos(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$
(11) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(12) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(13) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(14) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}d\theta$
(15) $S = \int_a^b 2\pi(x \text{ or } y)ds$
(16) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(17) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(18) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(19) $A = \int_a^b \frac{1}{2}r^2 d\theta$
(20) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(21) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$