MATH 265
FALL 2008
EXAM 2

1. (5 pt) Find all critical numbers and classify local extrema for the function $f(x, y) = x^3 - y^2 + xy$.

2. (5 pt) Find the maximum and minimum values of the function $g(x, y) = x^4 - 2x^2y^2 + y^3$ on the triangle with vertices $(0, 0), (4, 0)$, and $(4, 8)$.

3. (5 pt) Show that every normal line to the sphere $x^2 + y^2 + z^2 = R^2$ passes through the origin.

4. (5 pt) Let $z = f(x, y)$, $x = r\cos(\theta)$, and $y = r\sin(\theta)$. Show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2} + \frac{1}{r}\frac{\partial z}{\partial r}.$$

5. (5 pt) Consider the function $f(x, y, z) = \sqrt[n]{(ax + by + cz)^2}$. Find the directional derivative of this function in the direction of the vector $\langle -2, 3, -6 \rangle$. In what direction does this function increase most rapidly?

6. (5 pt) Consider the ellipsoid obtained by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the $y-$axis. Find the volume of the largest cylinder (with respect to volume) that can be put inside this ellipsoid.

7. (5 pt) The circular paraboloid $z = x^2 + y^2$ and the plane $x + y + 2z = 2$ intersect in an ellipse. Find the point(s) on this ellipse that minimize and maximize distance from the origin.