MATH 265
FALL 2009
EXAM 3

1. Consider the function $f(x,y) = x^4 + 2y^2 - 8xy$.
   a) (5 pts) Find all critical points of $f(x,y)$.
   b) (5 pts) Classify the critical points that you found in part a).

2. Once again, consider the function $f(x,y) = x^4 + 2y^2 - 8xy$. In this problem we restrict our attention to the domain enclosed by the parabola $y = \frac{1}{2}x^2$ and the line $y = 8$ (including the boundary). The goal is to find the maximum and minimum values of the function on this domain.
   a) (5 pts) What $x-$values need to be checked for maximum and minimum values of the function along the top (horizontal line)?
   b) (5 pts) What $x-$values need to be checked for maximum and minimum values of the function along the "bottom" (parabolic part of the boundary)?
   c) (5 pts) How many interior points are candidates and need to be checked?
   d) (5 pts) Describe the process you would take in finding the maximum and minimum values for this function on the domain (given the information from the first three parts of this problem)?

3. Consider the functions $f(x)$ and $g(y)$ and suppose that $f(x)$ has a local extreme value at $x = a$ and $g(y)$ has a local extreme value at $y = b$. You may assume that $g''(b) \neq 0$ and $f''(a) \neq 0$.
   a) (5 pts) Show that if $f(x)$ has a local maximum at a and $g(x)$ has a local maximum at b then $F(x,y) = f(x) + g(y)$ has a local maximum at (a,b).
   b) (5 pts) Show that if $f(x)$ has a local maximum at a and $g(x)$ has a local minimum at b then $F(x,y) = f(x) + g(y)$ has a saddle point at (a,b).

4. (5 pt) Use Lagrange multipliers to find the maximum and minimum values of $f(x,y,z) = ax + by + cz$ (a, b, c not all 0) on the sphere $x^2 + y^2 + z^2 = R^2$.

5. (5 pt) Find the following limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^3 + 2y^3}$$

or show that the limit does not exist.

6. (5 pt) Find the directional derivative of the function $f(x,y,z) = xy - z^3$ at the point $(2,1,3)$ in the direction of the vector $\langle -1, 1, 2 \rangle$. In what direction at this point is the directional derivative maximal?

7. (5 pts) Consider the the function $w = F(x,y,z), x = u_1(s,t), y = u_2(s,t)$, and $z = u_3(s,t)$. Find $\frac{\partial w}{\partial t}$.