MATH 265
FALL 2009
EXAM 4

1. Consider the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
   a) (5 pts) Set up a double integral to find the area enclosed by this ellipse.
   b) (5 pts) Now transform the above integral via the transformation $u = \frac{x}{a}, v = \frac{y}{b}$ (that is, find the Jacobian, integral, and the new region of integration).
   c) (5 pts) Evaluate the integral to find the area enclosed by the ellipse.

2. The goal of this problem is to evaluate the integral

$$\iiint_E z^2 dV$$

where E is the portion of the upper half of the sphere $x^2 + y^2 + z^2 = R^2$ that is between the cones $x^2 + y^2 = 3z^2$ and $x^2 + y^2 = \frac{1}{3}z^2$.
   a) (5 pts) Find the equation, in spherical coordinates of the cone $x^2 + y^2 = az^2$.
   b) (5 pts) Use spherical coordinates to evaluate the integral above.

3. (5 pts) Show that the moment of inertia of a sphere (uniform density given by ρ and radius R) about the $z-$axis is given by $I_z = \frac{2}{5}MR^2$ where M is the mass of the sphere.

4. Consider the integral

$$\iint_R \sin(\frac{x+2y}{x-2y}) dA$$

where R is the triangle with vertices $(0,0), (2,-1), (4,0)$.
   a) (5 pts) Consider the transformation $u = x - 2y$ and $v = x + 2y$. Solve these equations x and y.
   b) (5 pts) Find the Jacobian of the transformation.
   c) (5 pts) Find the new region of integration (in the uv plane).
   d) (5 pts) Evaluate the integral.