MATH 265
FALL 2009
EXAM 5

1. (5 pts) Let n be a positive integer and C_n be the circle $x^2 + y^2 = R^2$ traversed n times ('round and 'round n times counterclockwise). Evaluate

$$\oint_{C_n} \frac{x}{x^2+y^2}dy - \frac{y}{x^2+y^2}dx.$$

2. (5 pts) Let C be the closed loop (oriented positively) consisting of the right half of the circle $(x-4)^2 + y^2 = 4$, the left half of the circle $(x+4)^2 + y^2 = 4$, and the lines $y = 2$ and $y = -2$ $(-4 \le x \le 4)$. Evaluate the integral

$$\oint_C (2y\cos^2(x) + 2y\sin^2(x))dx - (3x + y^3 e^{y^3})dy.$$

3. (5 pts) Let C be a piecewise-smooth, closed path (beginning and ending at the point (a, b)) and let f and g be differentiable functions of y. Evaluate

$$\oint_C f(y)dx + (xf'(y) + g(y))dy.$$

4. Consider the vector field $\mathbf{F} = \langle ax, by, cz\rangle$.

a) (5 pts) Compute curl($\mathbf{F}$).

b) (5 pts) Find a function $f(x, y, z)$ such that $\nabla f = \mathbf{F}$.

c) (5 pts) Evaluate $\oint_C \mathbf{F}\cdot\mathbf{n}ds$ where C is the circle of radius R centered at the origin (oriented positively).

5. (5 pts) Suppose that f and g have continuous second partial derivatives. Show that

$$\text{div}(\nabla f \times \nabla g) = 0.$$