MATH 265
FALL 2009
FINAL EXAM

1. Consider the vectors $\overrightarrow{a} = \langle 2, -1, 3\rangle$ and $\overrightarrow{b} = \langle 1, 1, -4\rangle$. Find the following.
   a) (5 pts) $\overrightarrow{a} \cdot \overrightarrow{b}$.
   b) (5 pts) $\overrightarrow{a} \times \overrightarrow{b}$.
   c) (5 pts) The angle between $\overrightarrow{a}$ and $\overrightarrow{b}$.

2. (5 pts) Find the maximum and minimum values of $f(x, y) = x^2 + 5y^2 + 2xy - 8x - 16y$ on the triangle bounded by the lines $x = 4$, $y = 0$, and $y = x$.

3. (5 pts) A box with an open top and rectangular bottom is to be constructed from A square feet of material. Show that the maximum volume is obtained when the base of the box is a square.

4. Consider the vector field $\mathbf{F} = \langle 2x + yz^2, 1 + xz^2, 2xyz\rangle$.
   a) (5 pts) Compute div($\mathbf{F}$).
   a) (5 pts) Compute curl($\mathbf{F}$).
   b) (5 pts) Is ($\mathbf{F}$) the gradient of a potential function? If so, find a function $f(x, y, z)$ such that $\nabla f = \mathbf{F}$.

5. (5 pts) Find the volume of the portion of the cylinder $x^2 + y^2 = 1$ that lies between the planes $z = 6 + x + y$ and $z = -2 - x - y$.

6. (5 pts) Find the line of intersection of the planes $x + y - z = 2$ and $x - y - z = 10$.

7. Consider the portion of the surface $z = xy$.
   a) (5 pts) Find the tangent plane to this surface at the point (2,3,6).
   b) (5 pts) Find the surface area of the portion of this surface lying above the part of the circle $x^2 + y^2 = R^2$ in the first quadrant.

8. Consider the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
   a) (5 pts) Set up a double integral to find the area enclosed by this ellipse.
   b) (5 pts) Now use the coordinate change $x = ar\cos(\theta)$ and $y = br\sin(\theta)$ to evaluate the integral (do not forget to compute the Jacobian).

9. (5 pts) Get stoked and evaluate $\iint_S \text{curl}\,\mathbf{F} \cdot d\mathbf{S}$ where S is the hemisphere $x^2 + y^2 + z^2 = 16$, $z \geq 0$ oriented positively, and $\mathbf{F}(x, y, z) = \langle 2y\cos(z), e^x \sin(z), xe^y\rangle$.

10. (5 pts) Our last divergence will be to compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$ over the surface $y^2 + z^2 = 1$, $-2 \leq x \leq 2$. Here $\mathbf{F}(x, y, z) = \langle 3xy^2 + z^2, x\cos(z^2), z^3 - xy\rangle$