MATH 270
SUMMER 2007
HOMEWORK 1

First a word about the homework problems in general. You will find that some of them are worth 3 points (usually for the grader(s)) and some of them worth 5 (usually mine to grade). Be sure and attempt them all, and be sure to communicate clearly in the English language (remember, part of this course is learning to communicate your ideas effectively and points may be docked for lack of clarity). Homework is due at the beginning of class on the assigned day.

Due Friday June 15, 2007.

1. (5 pt) Give an example of a statement (with a subject and a predicate that is not a question or a command) that is neither true nor false (and explain why your example works).

2. (5 pt) You find yourself on a desert island. There are two types of natives who live on the island, the Truth-Tellers (who always tell the truth) and the Liars (who always lie). Traveling on the island, you come to a fork in the road. One way leads to a McDonalds (you are very hungry) and the other leads to a a K-Fed Concert (believe me, you do not want to go there). At the fork in the road is a native (you do not know from which tribe, however). You are permitted but one question to ask her. Formulate a question that you can ask the native that will direct you where to go and prove that your answer works.

3. (5 pt) **Theorem:** The money you have is equal to the money that you need.
Proof: Let M be the money that you have and let N be the money that you need. Consider the average of the two $A = \frac{N+M}{2}$. So we have

$$2A = M + N$$

and so multiplying both sides of the equation above by $M-N$ we get $2AM-2AN = M^2-N^2$. Equivalently, we can write $N^2-2AN = M^2-2AM$. We add A^2 to both sides to get $N^2-2AN+A^2 = M^2-2AM+A^2$. Factoring these equations we have:

$$(N-A)^2 = (M-A)^2.$$

Taking the square root of both sides, we see that $N-A = M-A$ and hence $M = N$. $\diamondsuit$
Criticize this proof or rethink your life!

4. (5 pt) Suppose that I have an infinite number of numbered ping pong balls (one for every positive integer) and a very large bag. At one minute to midnight, I balls numbered 1-10 in the bag and remove ball 1. At half a minute to midnight I put balls 11-20 in the bag and remove ball 2, at a third of a minute to midnight I put in balls 21-30 and remove ball 3... Assuming that I can continue this process indefinitely, how many balls are in the bag at the stroke of midnight? How many are left at midnight if I do the same thing as before, but this time I remove ball 10 at 1 minute to midnight, ball 20 at a half a minute to midnight, ball 30 at a third of a minute to midnight etc.?

5. (5 pt) Show that if n is any integer, then n^3-n is divisible by 6.